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On the Market for Venture Capital

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We propose a theory of the market for venture capital that links the excess return to venture equity to the scarcity of venture capitalists (VCs). High returns make the VCs more selective and eager to terminate nonperforming ventures because they can move on to new ones. The scarcity of VCs enables them to internalize their social value, and the competitive equilibrium is socially optimal. Moreover, the bilaterally efficient contract is a simple equity contract. We estimate the model for the period 1989–2001 and compute the excess return to venture capital, which turns out to be 8.6 percent. Finally, we back out the return of solo entrepreneurs, which is increasing in their wealth and ranges between zero and 3.5 percent.

I. Introduction

This paper develops a theory of the market for venture capital. We model not only the contractual relationship between entrepreneurs (Es) and venture capitalists (VCs) but also the market for such contracts. Several predictions of our model are consistent with empirical observations relating to firm financing, including the fact that VC-backed firms are of-

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ten worth more at initial public offerings (IPOs) and that founders of companies appear to be more patient than VCs. We estimate our model using a data set that covers VC-financed companies created between 1989 and 1993. We estimate the excess return to venture capital to be 8.6 percent. While high, this estimate is lower than the private-equity returns data imply for funds of the 1989–93 vintage. One feature of our model is particularly useful: It enables estimation of the returns of solo entrepreneurs using only data on VC-backed companies. We thereby find that the excess returns of solo entrepreneurs range between zero and 3.6 percent.

We consider a setup involving entrepreneurs and venture capitalists. Each E possesses a project and some amount of wealth, which varies across Es. To succeed, a project requires a continuous flow of investment as well as continuous effort exerted by the E. Both the project return and the time at which a project succeeds are random. After an initial setup cost is incurred, the return of a given project is observed, but no information is revealed about how long it will take for the project to succeed. Motivated by data, the hazard of the success time is assumed to first rise and then decrease. That is, after a certain point, agents become increasingly pessimistic about a quick success. The Es with little wealth are unable to invest in their projects for the optimal length of time, and they benefit from external funding. Each VC has unlimited wealth and can enter into a contractual relationship with at most one E at a time.[1] A contract specifies an initial transfer and a rule for sharing the return. However, neither the investment of a VC nor the effort of an E is contractible. We are particularly interested in the case in which VCs are scarce relative to Es.

We investigate our model's unique stationary equilibrium. We show that when VCs are scarce, their expected payoff in equilibrium is positive. That is, VCs earn an excess return on their investments. The equilibrium behavior of agents can be characterized by two types of decisions. First, Es must decide whether to go solo or seek VC financing, which gives rise to the following trade-off. On the one hand, since Es have limited wealth, going solo might prevent an E from financing her project up to the optimal termination time. On the other hand, if an E contracts with a VC, she must forfeit part of the project's return. We show that, in general, the set of wealth levels of those Es who are VC-backed in equilibrium is an interval. An E whose wealth lies above this interval will prefer to go solo. An E with wealth below the interval is so poor that she is unable to finance even a fraction of the setup cost. This makes it impossible for a VC to contract with such an E and realize his equilibrium payoff.

The second important decision agents must make involves specifying a termination time for the project, should it remain unsuccessful. The

[1] In practice, the general partners (GPs) of a venture fund provide the screening whereas funding comes from the limited partners (LPs). The VC in our model performs both functions.

termination decision of a solo E is quite simple: she terminates either because she runs out of money or because the marginal benefit of investing in the project reaches its marginal cost. The termination decision of a VC is more complex. Since a VC always has the option of moving on to a new company, attending to a nonperforming company entails for him a forgone-earnings cost that an E does not face. Hence, the first-order condition defining the VC's optimal termination time takes into account the fact that his total cost of financing a project is the sum of the capital investment and the opportunity cost of staying with a project rather than moving on to something new.

The opportunity cost faced by VCs plays an important role in explaining the differences between solo and VC-backed companies. This higher cost imposes a higher quality hurdle. That is, a VC's cutoff, the lowest return for which he is willing to finance a project, is higher than that of a wealthy solo E. This selection effect raises the IPO values of VC-backed firms above those of other firms, consistent with empirical studies (see Megginson and Weiss 1991; Hochberg 2012). The novelty of our explanation is that it does not rely on an assumption that VCs somehow increase the quality of the firm they finance. Instead, it is based on a simple selection argument: The opportunity cost faced by a VC induces him to terminate faster than a wealthy E would, and to stay alive in a VC's portfolio, a project must have a higher expected quality. This is in accord with the results of Sørensen (2007), who finds that the bulk of the observed positive association between VC quality and project quality is due not to direct VC influence on the payoff but to sorting. The VCs in fact do appear to be less patient than solo Es. Jones and Rhodes-Kropf (2004) report that founders are more attached to and more patient with their projects than VCs who behave as if their discount rates were higher.

Our model describes an environment in which efficiency is achieved via simple equity contracts between VCs and Es, despite the fact that an E's effort is unobservable.[2] Such contracts are commonly used in practice, as Kaplan and Strömberg (2003) document. Using equity contracts is optimal in our setting for the following reason. Bilateral efficiency requires that the optimal termination time maximizes the total benefit of financing a project net of total costs. Both the VC and the E incur only a fraction of the total cost, since the VC invests the capital and the E exerts the effort. If the equity contract specifies a sharing rule that allocates a fraction of the benefit to each party according to his proportional contribution to the cost, then maximizing the objective of each agent is

[2] In the search-matching models of Inderst and Müller (2004) and Michelacci and Suarez (2004), Nash bargaining divides rents between the VC and E. In these models, the nongeneric "Hosios condition" must hold for the equilibrium to be efficient. In contrast, our model generates efficiency on an open set of all parameter values.

equivalent to maximizing total surplus, scaled down by a constant. Since scaling has no effect on the solution, each agent terminates the project efficiently. Of course, equity contracts might not eliminate some other agency problems that are not present in our model. Bergemann and Hege (2005), for example, assume that Es can divert VC funds for private consumption. In their model, equity contracts do not restore efficiency, and projects are terminated too early because the Es can extract rent as a result of the nonverifiability of their investment decisions.

Finally, we calibrate our model, showing that an eight-parameter example provides a fairly good fit to our data set, which covers approximately 1,000 VC-supported companies founded between 1989 and 1993. The model targets the cross-sectional distribution of exit values, the flow costs of investment, the flow of net revenue as a function of company age or the "*J* curve," and the essential properties of the distribution of waiting times until "success" (defined as IPO or private sale) or failure. In addition, our numerical exercise accomplishes two goals. First, we estimate an excess return to venture capital of 8.6 percent, which is not surprising given the high returns earned by private-equity funds begun in the 1989–93 period (Cambridge Associates 2011; Hochberg, Ljungqvist, and Vissing-Jorgensen, forthcoming).[3] Second, our model generates a distribution of Es' returns, which are not included in our data. In our model, projects are ex ante identical. The only reason for an E to seek VC backing is that she is wealth constrained. From our data on VC-backed companies, we are able to back out all necessary parameters of the distribution of the projects in order to estimate the return to solo entrepreneurs. We estimate the profit of a wealth-unconstrained solo E to be 3.6 percent. Constrained solo Es earn lower but still positive excess returns.

Several features of our model are related to aspects of existing models. First, in our model a VC is better equipped to evaluate projects than a bank. Ueda (2004) studies an E's choice between bank financing and VC financing, and, as in our model, a VC is better able to assess projects than banks. However, the VC can also use information revealed to him to set up a competing business and thereby reduce the value of an E's project.

Second, in our model a VC might abandon one project so that he can move on to another. In Holmes and Schmitz (1990), managers of projects sometimes terminate existing projects in order to start new ones; the trade-off is similar to the one faced by VCs in our model.

Third, in our model the rents enjoyed by VCs are determined by the supply of VCs relative to the number of Es, in conjunction with the dis-

[3] Depending on the study and the time period, a wide range of α's (excess returns) have been estimated for venture capital: Cochrane (2005) estimates α at over 30 percent, Ljungqvist and Richardson (2003) and Kaplan and Schoar (2005) argue that it is around 5 percent, and Hall and Woodward (2007) estimate it at around 2 percent.

tribution of wealth among the population of Es. Free VCs face no delays in matching with Es, but the maturation of a project is a multiperiod process during which information about the project's quality may come to light. Michelacci and Suarez (2004) have a similar model, but they introduce free entry of Es who, in contrast to our model, always require outside financing. They also introduce a matching friction, and the rents of a project are divided by Nash bargaining between the E and the VC. Inderst and Müller (2004) drop the Nash-bargaining assumption; in their model a project lasts one period, and their optimal contract entails an equity-sharing rule.

Finally, in this paper we investigate multiperiod optimal contracting with no commitment, and the literature on this topic is vast. For example, we have already mentioned Bergemann and Hege (2005), who study multiperiod contracting between a single VC and a single entrepreneur, with their outside options taken as given. In their model, an E can divert the invested funds to private consumption. The authors show that, in order to provide the E with proper incentives, the optimal contract specifies a decreasing stream of investment funds. As a result, the project is supported for less time than would be socially efficient. While in Bergemann and Hege's study incentives cause early and inefficient terminations of VC-backed projects, the VC's high opportunity cost results in early, yet efficient, terminations in our model.

Plan of the paper.—Section II describes the model, and Section III characterizes the unique stationary equilibrium. Section IV discusses the relation between equilibria and welfare. Section V presents an example, fits it to data, and discusses the implications. Section VI concludes the paper. Online Appendix A contains the proofs, Appendix B describes the data, Appendix C describes the solution algorithm and the estimation procedure in detail, Appendix D reports several simulations, and Appendix E discusses robustness.

II. Model

Consider a measure x of infinitely lived VCs, each able to raise a sufficient amount of money at rate r. There is also an inflow of projects at a rate normalized to one. Each project is in the possession of a different E. The Es cannot borrow and have initial wealth w, which is distributed according to the cumulative distribution function (CDF) Ψ. An E can have at most one project in her lifetime.

A. *Projects*

To succeed, a project requires an immediate payment of a start-up cost C, and from the moment of start-up until the project yields a payoff, it also requires k units of continuous investment and a units of continuous effort on the part of E. A project can be undertaken by an E together with a VC or by the E alone, in which case she must rely only on her own wealth. A VC can finance at most one project at a time. The project yields payoff π at time τ, where π and τ are random variables independent of each other and π has a finite mean. Let G and F be the CDFs of π and τ, respectively, and let g and f denote the corresponding densities. Both G and F have support in $\mathbb{R}_+$. Assume that the hazard rate $f/(1-F) = h$ is bell shaped: it increases up to the modal age, τ_m, then decreases. Agents' optimism about a quick realization of τ initially grows, but after age τ_m they become increasingly pessimistic.[4] Neither party knows π or τ ex ante, but their distributions are common knowledge. If the project is a solo venture, E learns π after incurring the cost C. If the venture is VC-backed, π becomes known to both parties after the cost C is incurred. The signal, π, is treated as being perfect, but one could, with minor changes in the analysis, assume it to be only a noisy estimate of the profit. On the other hand, no information about τ is observed. If the project ever suffers from either underinvestment or a lack of effort for at least one period, it will never yield a positive payoff. The expected value to developing projects is assumed to be positive. That is, if financing decisions regarding the project are made optimally, it yields a positive payoff in expectation.

B. *Preferences*

All agents are risk neutral and discount the future at the rate r, which is equal to the risk-free interest rate at which agents can save. The VC maximizes the expected discounted present value of his net income. The E maximizes the expected discounted present value of her net income minus the disutility associated with the effort she exerts.

C. *Contracting*

Feasible contracts.—The contract a VC can offer consists of two numbers: (p, s). The number p is a lump sum the VC pays E immediately upon signing the contract. The number $s\ (\geq 0)$ specifies the share of the payoff kept by the E if the project succeeds. If the project yields payoff π, E keeps $s\pi$ and the VC receives $(1-s)\pi$. The payment p and the sharing rule s are enforceable. On the other hand, neither E's effort nor the VC's ongoing investments are contractible. After the transfer p is paid, the arrangement is a pure equity contract. We could in theory allow for more complicated contracts in which s depends on τ and π. However, we

[4] Our results would follow more easily if the hazard were to decline monotonically throughout. The bell-shape assumption conforms better to the empirical success hazard plotted in panel 4 of fig. 5 below.

will show that for parameter values of our interest, we can restrict attention to this simple contract type without loss of generality.

Timing of the contractual relationship.—If E signs the contract (p, s), she receives a payment p from the VC immediately. Then E finances the start-up cost C, and both parties learn the value of π.[5] At each date, if the payoff has not yet been realized, E has to decide whether or not to exert effort and the VC has to decide whether or not to invest. We assume that both parties can observe the history of investments and effort when making these decisions.[6] (We show that our equilibria remain valid regardless of whether the parties can observe each other's support for the project.) If the VC decides to no longer invest in the project, he is free to devote his time to another venture. If E decides to stop exerting effort, she exits the market. If the project succeeds, the two parties share the surplus according to s and their involvement comes to an end. The VC seeks a new match, and E leaves the market.

D. *Market Structure*

There is a market for each contract (p, s). Each VC who is not in a contractual relationship and each E who seeks VC backing must decide which market to enter.[7] Suppose that at time t a measure n of VCs and a measure m of Es enter the market corresponding to contract (p, s). Then $\min\{n, m\}$ pairs of VCs and Es are randomly matched and sign the contract (p, s). A VC who is not matched can decide to either stay in the market or choose a different one. If an E is unmatched, she is forced to leave the market and abandon her project.[8] When an E first has an idea for a project, she must decide whether to abandon it outright, seek VC backing, or go solo, that is, implement her project on her own. This decision is irreversible.

E. *Banks*

In our model, banks guarantee a risk-free interest rate, but they do not finance projects, as they face two disadvantages with respect to VCs. First, banks lack the expertise of VCs and Es; banks learn π only on the date

[5] We interpret p as the contribution the VC makes towards financing C.

[6] So as to avoid coordination problems, we assume that we have a continuous-time limit of discrete periods in each of which the VC moves first and the E moves second. This rules out the no-effort equilibrium.

[7] Again, in order to avoid coordination problems, we assume that VCs make these decisions first and Es observe the measure of VCs in each of these markets before they make their own ones.

[8] This assumption is made only for convenience. We will show that an E who does not get matched in a market would, in any equilibrium, have made zero even if she could sign a contract.

the project succeeds, τ, and not before. Second, banks lack the ability to monitor borrowers' behavior, while VCs can ensure that Es do not divert investment toward private consumption. In fact, banks never offer contracts to Es; any borrower could claim to be an E while using the funds for personal use, leaving the bank with negative profits.

F. Equilibrium

The strategy of an agent specifies the market he or she enters and, after a contract is signed and π is revealed, the time at which support for the project will be terminated. We restrict attention to *stationary* equilibria, in which decisions do not depend on calendar year or history. An E's decision regarding which market to enter depends only on her wealth, w. We characterize the unique competitive equilibrium for each value of x, the measure of VCs. However, we are particularly interested in the case in which x is small, and the empirical part of the paper focuses on this case.

III. Analysis

We first analyze the termination decisions of solo Es and compute the threshold level of wealth above which an E is better off going solo than abandoning her project. We then examine the termination decisions regarding VC-backed projects and characterize equilibrium contracts. Finally, we identify the set of Es who are VC-backed as a function of the equilibrium value of a VC.

A. The Solo Entrepreneurs

Termination problem.—A solo entrepreneur might run out of money and have to terminate her project. So, E will optimally defer all her consumption until the project is completed.[9] Consider an E with wealth w and let w_t denote E's wealth at time t if her project has not yet succeeded. At every instant of time, E must invest k and receives interest on w_t. So, the evolution of w_t is described by the differential equation $\dot{w}_t = rw_t - k$. The initial condition is $w_0 = w - C$ because the cost C must be incurred immediately. Solving for w_t gives

$$w_t = \frac{k}{r} + \left(w - C - \frac{k}{r}\right)e^{rt}.$$

[9] Entrepreneurs save more in order to get around liquidity constraints; see Basaluzzo (2006) and Buera (2009).

For each $w \geq C$, let $\tau(w)$ be the date at which the entrepreneur's wealth is depleted. The date $\tau(w)$ is obtained by solving for t in the equation $w_t = 0$:

$$\tau(w) = \begin{cases} \dfrac{1}{r}\ln\left[\dfrac{k}{k - r(w - C)}\right] & \text{if } w < \dfrac{k}{r} + C \\ \infty & \text{if } w \geq \dfrac{k}{r} + C. \end{cases} \tag{1}$$

The unconstrained entrepreneur.—Suppose that $w \geq k/r + C$; that is, E has enough money to finance her project forever. Since new ideas occur only to new entrepreneurs, E's continuation value is zero if she terminates the project prior to success. If E's project is worth π, the payoff she collects by supporting it until time T is[10]

$$\int_0^T e^{-rt}\{\pi f(t) - (a + k)[1 - F(t)]\}\,dt$$

$$= \int_0^T \left[\pi - \frac{a + k}{h(t)}\right] e^{-rt} f(t)\,dt,$$

where $h = f/(1 - F)$. Therefore, for each π, she solves the following maximization problem:

$$\max_T \int_0^T \left[\pi - \frac{a + k}{h(t)}\right] e^{-rt} f(t)\,dt.$$

The solution of this problem, $T^S(\pi)$, is depicted in figure 1. Suppose that the project yields a payoff π_1 and the optimal termination time, $T^S(\pi_1)$, is interior. Then $T^S(\pi_1)$ satisfies both the first-order condition $a + k = \pi_1 h(T^S(\pi_1))$ and the second-order condition $h'(T^S(\pi_1)) < 0$. Note that if $\pi_2 > \pi_1$, then $T^S(\pi_2) > T^S(\pi_1)$. Finally, let $\pi^S_{\min}$ denote the smallest realization of π that the unconstrained entrepreneur should support. Then

[10] To see this, note that if $t > T$, the project exited already and no investment is made. If $t \leq T$, a proportion $F(t)$ of the projects succeeded, and only a measure $1 - F(t)$ of them need investment. Total cost then is

$$\int_0^T e^{-rt}(a + k)[1 - F(t)]\,dt = \int_0^T e^{-rt}(a + k)\frac{1 - F(t)}{f(t)}\,dF(t)$$

$$= \int_0^T e^{-rt}\frac{a + k}{h(t)}\,dF(t).$$

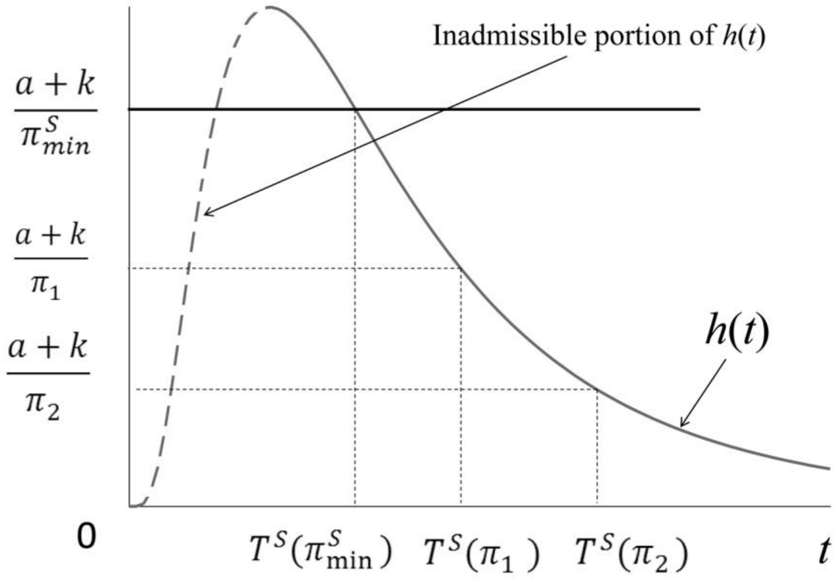

FIG. 1.—The determination of $T^S(\pi)$

$$T^S(\pi) = \begin{cases} h^{-1}\left(\dfrac{a+k}{\pi}\right) & \text{if } \pi > \pi^S_{\min} \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

The wealth-constrained entrepreneur.—Suppose now that $w < k/r + C$. After E pays the cost C and learns π, the expected value of the project is

$$q(\pi, w) \equiv \max_{T\in[0,\tau(w)]} \int_0^T \left[\pi - \frac{a+k}{h(t)}\right] e^{-rt} f(t)\,dt. \qquad (3)$$

The solution $T^S_w(\pi)$ either is equal to zero or is found by solving for t in the first-order condition:

$$\pi h(t) \geq a + k,$$

which holds with equality whenever $0 < T^S(\pi) \leq \tau(w)$. Therefore, if the project is worth pursuing after incurring C, then the solution is $\min(\tau(w), T^S(\pi))$, and if not, it is zero. If $\pi^S_{\min}(w)$ denotes the smallest payoff for which it is worth providing support initially, then

$$T_w^S(\pi) = \begin{cases} \min\left\{\tau(w),\ h^{-1}\left(\dfrac{a+k}{\pi}\right)\right\} & \text{if } \pi > \pi^S_{\min}(w) \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Who should start a project?—The expected surplus, after incurring the cost C, of a solo entrepreneur with wealth w is

$$Q^S(w) = \int q(\pi, w)\, dG(\pi).$$

The next lemma characterizes some important features of the curve Q^S.

LEMMA 1. (i) For $w < k/r + C$, $\partial Q^S/\partial w > 0$. (ii) For $w \geq k/r + C$, $\partial Q^S/\partial w = 0$ and $Q^S(w) > C$.

Part i holds because an additional dollar, rather than being consumed, can be used by a budget-constrained E to prolong her project and to generate a positive surplus. To see part ii, note that an E with $w > k/r + C$ can support her project for as long as it is privately optimal. An additional dollar would then not generate an excess return, so in this region the slope of Q^S in figure 2 is zero. Since the expected value of a project is positive by assumption, $Q^S(w) > C$ whenever $w > k/r + C$.

Going solo or abandoning the project.—Suppose that VC backing were not an option. The payoff to going solo with wealth w would be $w - C + Q^S(w)$. We explain why this payoff must appear as depicted in figure 2. The payoff is not defined for $w < C$ because E would be unable to pay C. At $w = C$, it is zero because, after paying C, E would have no money left to invest in the project. For larger values of wealth the slope exceeds unity, as shown in part i of lemma 1. At $w = k/r + C$, $w - C + Q^S(w) = w + \sigma$, where $\sigma > 0$ because $Q^S(k/r + C) > C$ by part ii of lemma 1. The intermediate value theorem implies that there is a unique w^* at which $Q^S(w^*) = C$. An E with w prefers going solo over abandoning her project if and only if $w \geq w^*$.

B. VC Backing

In every stationary equilibrium, the expected payoff of a free VC, V^*, is constant across time and across VCs. This payoff determines a VC's opportunity cost of supporting a project and plays an important role in termination decisions. We refer to V^* as the *value* of a VC. The following subsection identifies the termination decisions for VC-backed projects, the equilibrium contracts, and the set of VC-backed Es as functions of V^*.

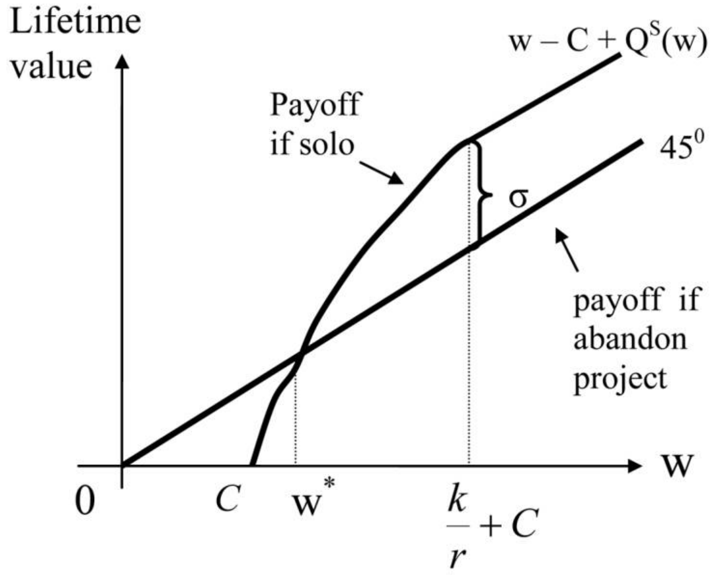

FIG. 2.—The payoff of solo Es

1. Termination Problems after a Contract Is Signed

We now analyze the incentives of each agent to support a project after the contract (p, s) is signed and both parties learn the value of π, if the value of a VC is V^*.

Entrepreneur.—If E believes that her project will be financed by the VC until time $\overline{T}$ and that she will get $s\pi$ if the project is successful, she solves the following problem:[11]

$$\max_{T \le \overline{T}} \int_0^T \left[s\pi - \frac{a}{h(t)} \right] e^{-rt} f(t)\, dt.$$

Let $T^E(\pi, s, \overline{T})$ denote the solution to this problem. If π is below a certain cutoff, denoted by $\pi^E_{\min}(s, \overline{T})$, E never exerts effort, and hence $T^E(\pi, s, \overline{T}) = 0$. Otherwise, the solution satisfies the following first-order condition:

$$h(T^E(\pi, s, \overline{T})) \ge \frac{a}{s\pi}. \tag{5}$$

[11] Since a project is terminated if either the VC or the E terminates it, the optimal termination time of E can be assumed to be smaller than $\overline{T}$.

The local second-order condition, which is also the sufficient condition, is $h'(T^E(\pi, s, \overline{T})) < 0$. Note that because of the bell-shaped hazard assumption, $h(T^E(\pi, s, \overline{T})) > a/s\pi$ implies $T^E(\pi, s, \overline{T}) = \overline{T}$ whenever $\pi \geq \pi^E_{\min}(s, \overline{T})$.

VC.—Suppose that the value of a free VC is V^*. If the VC trusts E to support the project until time $\overline{T}$, then, after signing the contract, his maximization problem is

$$\max_{T \leq \overline{T}} \int_0^T \left[(1 - s)\pi + V^* - \frac{k}{h(t)}\right] e^{-rt} f(t)\, dt + e^{-rT}(1 - F[T])V^*.$$

Let $T^{VC}(\pi, s, V^*, \overline{T})$ denote the solution to this problem. If π is lower than a certain cutoff, denoted by $\pi^{VC}_{\min}(s, V^*, \overline{T})$, the VC does not invest in the project at all and $T^{VC}(\pi, s, V^*, \overline{T}) = 0$. Otherwise, the solution must satisfy the first-order condition

$$h(T^{VC}(\pi, s, V^*, \overline{T})) \geq \frac{k + rV^*}{(1 - s)\pi}. \tag{6}$$

The sufficient condition is again $h'(T^{VC}(\pi, s, V^*, \overline{T})) < 0$. If (6) holds with strict inequality, then $T^{VC}(\pi, s, V^*, \overline{T}) = \overline{T}$ whenever $\pi \geq \pi^{VC}_{\min}(s, V^*, \overline{T})$.

If the project is terminated at time $\overline{T}$, it must be that $\overline{T} = T^E(\pi, s, \overline{T}) = T^{VC}(\pi, s, V^*, \overline{T})$. The largest solution of this chain of equalities satisfies both (5) and (6), but it is also solved by any smaller termination time. However, all of the smaller solutions are due to coordination failure; that is, both parties would continue to invest in the project at $\overline{T}$ if they knew that the other party would do the same. Since we assume away coordination problems (see the Stackelberg assumption in n. 6), a VC-backed project with return π will be terminated at time

$$T(\pi, s, V^*) = \max\{T : T^E(\pi, s, T) = T^{VC}(\pi, s, V^*, T)\}. \tag{7}$$

Again, let $\pi^{VC}_{\min}(V^*, s)$ denote the cutoff below which a project is terminated immediately.

2. Equilibrium Contracts

We next describe the contracts that will be signed in equilibrium. We show that if E is sufficiently rich, the equilibrium contract will involve a sharing rule that maximizes the sum of the contracting parties' surpluses. If, on the other hand, E is poor, the equilibrium sharing rule favors the VC too strongly, and E will terminate the project while the joint surplus from investing is still positive.

We first characterize the sharing rule that maximizes the joint surplus. Let $Q^E(s, V^*)$ and $Q^{VC}(s, V^*)$ denote the continuation values of an E and a VC, respectively, after accepting the contract (p, s) and paying C. That is,

$$Q^E(s, V^*) = \int\int_0^{T(\pi,s,V^*)} \left[s\pi - \frac{a}{h(t)}\right] e^{-rt} dF(t) dG(\pi)$$

and

$$Q^{VC}(s, V^*) = \int\int_0^{T(\pi,s,V^*)} \left[(1-s)\pi + V^* - \frac{k}{h(t)}\right] e^{-rt} dF(t) + V^* e^{-rT(\pi,s,V^*)}[1 - F(T(\pi, s, V^*))] dG(\pi).$$

The relationship between the sharing rule and the joint surplus is stated in the next lemma.

LEMMA 2. (i) $\arg\max_{s\in[0,1]}[Q^E(s, V^*) + Q^{VC}(s, V^*)] = a/(a + k + rV^*)$. (ii) $\arg\max_{s\in[0,1]} Q^{VC}(s, V^*) \in (0, a/(a + k + rV^*))$.

To develop intuition for part i, note that if $s = a/(a + k + rV^*)$, then $T(\pi, s, V^*)$ satisfies both (5) and (6) with equality whenever $\overline{T}$ is sufficiently large. In addition, with $\overline{T} = T(\pi, s, V^*)$, both of these conditions are reduced to

$$h(T(\pi, s, V^*)) = \frac{a + k + rV^*}{\pi}, \tag{8}$$

which is just the first-order condition for joint surplus maximization. If $s < a/(a + k + rV^*)$, the joint surplus shrinks because E terminates the project prematurely. However, since the VC's share is larger, his continuation value is also larger in some cases, as stated in part ii.

The next lemma characterizes the equilibrium contracts.

LEMMA 3. Suppose that the equilibrium value of a free VC is V^* and an E with w is VC-backed. If $w \geq C + V^* - Q^{VC}(V^*, a/(a + k + rV^*))$, the contract this E signs is $s = a/(a + k + rV^*)$ and $p = Q^{VC}(V^*, s) - V^*$. Otherwise, $p = C - w$ and s is defined by $\max_{s\in[0,1]} Q^{VC}(V^*, s) = V^* + p$.

Since VCs compete in contracts, the equilibrium sharing rule is constrained efficient. If E has enough wealth, the sharing rule maximizes the sum of the continuation values and, hence, equals $a/(a + k + rV^*)$ by part i of lemma 2. The up-front payment p is determined by the fact that the VC's value should equal V^*. If $w < C + V^* - Q^{VC}(V^*, a/(a + k + rV^*))$, then E is unable to accept this contract because after signing it she is unable to cover C. In order for the VC to enter into a contract with such an E, he must increase the up-front payment p. The VC might be able to do so by offering a contract that specifies an s that is smaller than $a/(a +$

$k + rV^*$). The continuation value of a VC offering this s is higher than that derived from the sharing rule $a/(a + k + rV^*)$ by part ii of lemma 2, which compensates him for the larger p.

3. Who Should Get VC Backing?

This subsection identifies the wealth levels of those Es who get VC backing as a function of V^*. The key step involves characterizing the payoff a free VC would receive if he could make a take-it-or-leave-it offer to an E with wealth w. We denote this payoff by $V(w)$. We will show that the set of w values of VC-backed Es is an interval with end points determined by $V(w) = V^*$. That is, the equilibrium contracts signed by both the poorest and richest VC-backed Es are take-it-or-leave-it offers.

Suppose that a VC can always offer a take-it-or-leave-it contract to E with w and that it yields him a value of V. Then this contract solves the constrained maximization problem

$$\max_{(p,s)} Q^{VC}(V, s) - p, \tag{9}$$

subject to

$$C - p \le Q^{E}(V, s), \tag{9a}$$

$$C \le p + w, \tag{9b}$$

and

$$w + Q^{S}(w) - C \le w + Q^{E}(V, s) - (C - p), \tag{9c}$$

where (9a) guarantees that E at least breaks even, (9b) requires that E can pay C after receiving p, and (9c) ensures that E prefers signing the contract to going solo. Let $P(V)$ denote the value of this problem. Of course, a VC can always choose not to participate in the market, thereby receiving a value of zero. Hence, the value $V(w)$ is the unique solution to $V = \max\{0,\ P(V)\}$.

We next solve the VC's problem, ignoring (9b) and (9c), which leads to an upper bound on V^*. To this end, let $\overline{V}$ denote the value of the problem (9) subject to (9a).

LEMMA 4. The solution to (9) subject to (9a) is $(\bar{p}, \bar{s})$, where $\bar{p} = C(k + r\overline{V})/(a + k + r\overline{V})$ and $\bar{s} = a/(a + k + r\overline{V})$, so that $\bar{p} = (1 - \bar{s})C$. In addition, $C - \bar{p} < w^*$.

Note that if $w \geq C - \bar{p}$, then, after signing $(\bar{p}, \bar{s})$, E can pay the cost C and (9b) is satisfied. In addition, if $w \leq w^*$, then (9c) is also satisfied because E's payoff from going solo is negative (see fig. 2). To summarize, if $w \in [C - \bar{p}, w^*]$, then $(\bar{p}, \bar{s})$ satisfies (9a)–(9c) and, hence, $V(w) = \overline{V}$. On the other hand, if $w \notin [C - \bar{p}, w^*]$, then either (9b) or (9c) is binding and, hence, $V(w) < \overline{V}$.

Next, we characterize the curve $V(w)$ on the rest of its domain. Define $w_{\min} = \inf\{w : V(w) > 0\}$ to be the lowest value of wealth for an E such that the VC can still make a nonnegative payoff.

LEMMA 5. The function V is continuous. In addition,

i. $w_{\min} = 0$ or $V(w_{\min}) = 0$,
ii. V is strictly increasing on $[w_{\min}, C - \bar{p}]$,
iii. $V \equiv \overline{V}$ on $[C - \bar{p}, w^*]$,
iv. V is strictly decreasing on $[w^*, k/r + C]$, and
v. $V(w) = 0$ if $w \geq k/r + C$.

Part i identifies two cases that need to be considered, depending on the parameter values of the model. In the first case, the VC is able to make a positive profit by signing a contract with an E who is penniless $(w_{\min} = 0)$, and in the second case, a VC who contracts with an E with $w_{\min}$ obtains a value of zero $(V(w_{\min}) = 0)$. Figure 3 plots the function V for the case $w_{\min} > 0$.

We are now ready to derive the relation between the equilibrium value of a free VC, V^*, and the set of those Es who are VC-backed. If $V(w) > V^*$, an E with wealth w is VC-backed, for otherwise a VC can offer a contract

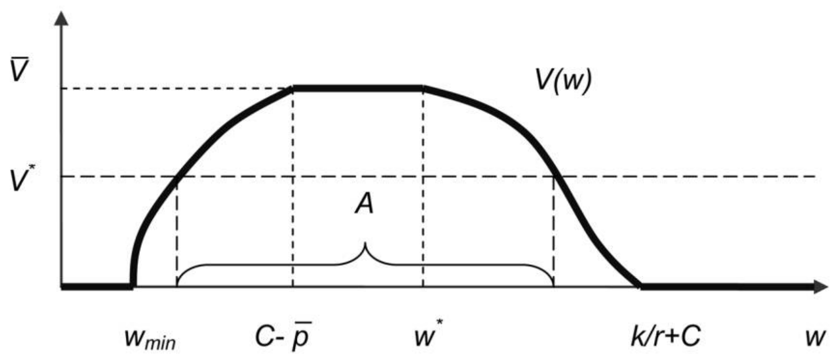

FIG. 3.—The function $V(w)$

that is accepted by E and provides the VC with a value greater than V^*. The flip side is that if $V(w) < V^*$, an E with wealth w does not receive VC financing. To summarize, if A is the set of VC-backed Es in equilibrium, then

$$\{w | w \geq w_{\min},\ V(w) > V^*\} \subset A \subset \{w | w \geq w_{\min},\ V(w) \geq V^*\}. \tag{10}$$

For each V^*, let $\mathcal{A}(V^*)$ denote the collection of those A's that satisfy (10). If $V \in (0, \overline{V})$, the set of VC-backed Es is essentially uniquely determined because there are only two Es who are indifferent between getting VC financing and taking their outside options; see figure 3. The remaining cases, where V^* is either zero or $\overline{V}$, are stated in the following lemma.

Lemma 6. (i) $A \in \mathcal{A}(0)$ if and only if $(w_{\min},\ k/r + C) \subset A \subset [w_{\min},\ \infty)$. (ii) $A \in \mathcal{A}(\overline{V})$ if and only if $A \subset [C - \bar{p},\ w^*]$.

IV. Equilibria and Welfare

This section characterizes equilibria and establishes a version of the first welfare theorem, which holds when the number of VCs is either small or large.

A. *Competitive Equilibria*

We show that there exists an equilibrium that is essentially unique; that is, each agent gets the same payoff across all equilibria. Recall that x denotes the measure of VCs.

Theorem 1. For all $x \in \mathbb{R}_+$, the value of a free VC, V^*, is uniquely determined. In addition, there exist $\underline{x}$, $\bar{x} \in \mathbb{R}_+$ $(\underline{x} < \bar{x})$ such that

i. if $x \leq \underline{x}$, then $V^* = \overline{V}$;
ii. if $x \in [\underline{x},\ \bar{x}]$, then $V^* \in (0,\ \overline{V})$; and
iii. if $x \geq \bar{x}$, then $V^* = 0$.

In each case, the set of VC-backed Es satisfies (10), and the equilibrium contracts are described by lemma 3. The termination decisions are defined by (2), (4), and (7).

As demonstrated in the previous section, V^* determines the termination decisions and the equilibrium contracts. We have also shown that the set of VC-backed Es, A, is an element of $\mathcal{A}(V^*)$ as it satisfies (10). In order to prove the theorem, we must explain how V^* and A are determined. To this end, we shall derive a market-clearing condition that requires the instantaneous measure of inflow of VCs to be the same as the outflow. We characterize both of these quantities as a function of V^* and

A. Finally, we show that this market-clearing condition has a unique solution for V^*.

Before proceeding, we first show that unless a free VC breaks even, he contracts with an E without delay. The reason is that if a VC waits after offering a contract, he could profitably deviate and avoid delay by offering a slightly larger up-front payment.

LEMMA 7. VCs do not wait in equilibrium unless $V^* = 0$.

Market-clearing condition.—VCs leave the market only when they sign contracts with Es. Therefore, the measure of the outflow of VCs must equal the measure of VC-backed Es. Let d_Ψ denote the measure induced by the CDF d_Ψ,[12] so the measure of outflow of VCs is just $d_\Psi(A)$.

VCs reenter the market when they stop financing projects because of success or termination. The measure of inflow of free VCs is the total number of VCs who support a project at a given point in time divided by the average duration of a VC-backed project. By lemma 3 the expected duration of a VC-backed project conditional on V^* and A is defined by

$$t(V^*, A) = \frac{\int_A \int_0^\infty \int_0^\infty \min(t, T(\pi, s(w, V^*), V^*)) f(t)\, dt dG(\pi)\, d\Psi(w)}{d_\Psi(A)}. \quad (11)$$

How many VCs support projects at a given moment? If $V^* > 0$, VCs do not wait (see lemma 7). Hence, the measure of the VCs financing a project is always x and the inflow of free VCs is $x/t(V^*, A)$. If $V^* = 0$, then VCs might wait in equilibrium, and all we can say is that the inflow of free VCs is at most $x/t(0, A)$. To summarize, the market-clearing condition requires that there exists an $A \in A(V^*)$ such that

$$d_\Psi(A) \begin{cases} \leq \dfrac{x}{t(0, A)} & \text{if } V^* = 0 \\ = \dfrac{x}{t(V^*, A)} & \text{if } V^* \in (0, \overline{V}]. \end{cases} \quad (12)$$

Existence and uniqueness.—We show that there exists a unique V^* that satisfies (12). Let $\Gamma(V^*, A)$ denote the numerator of the fraction on the right-hand side of (11), that is,

$$\Gamma(V^*, A) = \int_A \int_0^\infty \int_0^\infty \min(t, T(\pi, s(w, V^*), V^*)) \times f(t)\, dt dG(\pi)\, d\Psi(w). \quad (13)$$

[12] Formally, $d_\Psi(S) = \int_S 1 d\Psi$ for any measurable set S.

By (11), the market-clearing condition can be rewritten as

$$\Gamma(V^*, A)\begin{cases}\le x & \text{if } V^* = 0\\ = x & \text{if } V^* \in (0,\ \overline{V}].\end{cases} \tag{14}$$

Lemma 8 of online Appendix A establishes that as a correspondence of V^*, $\{\Gamma(V^*, A) | A \in \mathcal{A}(V^*)\}$ looks as depicted in figure 4.

Equipped with figure 4, we are ready to make an argument for the uniqueness of V^*. Define $\underline{x}$ and $\bar{x}$ to be $\Gamma(\overline{V}, [C - \bar{p},\ w^*])$ and $\Gamma(0, [w_{\min},\ k/r + C])$, respectively. If $x \le \underline{x}$, by (14) the equilibrium value of a VC must be $\overline{V}$. This case corresponds to x_1 in figure 4. As a consequence, the intersection of Γ and the horizontal line determines $d_\Psi(A)$. If $x \in (\underline{x}, \bar{x})$, the value of the VC lies in $(0,\ V^*)$ (see, e.g., x_2 in fig. 4). In this case, both V^* and A are pinned down. If $x \ge \bar{x}$, then $V^* = 0$. By (14), it is possible that x is larger than $\sup_{V^*, A \in \mathcal{A}(V^*)} \Gamma(V^*,\ A)$. In this case, free VCs must wait to contract with Es; see x_3 in figure 4.

Indeterminacy.—If $V^* = 0$, VCs support each E with $w \in (w_{\min},\ k/r + C)$ as well as some unconstrained Es $(w > k/r + C)$. While there are equilibria in which some VCs back some unconstrained Es, other equilibria exist in which the VCs wait and finance only constrained Es. Since $V^* = 0$ and VCs do not add value to a project owned by an unconstrained E, both the VC and the unconstrained E will be indifferent between entering and not entering into a contractual relationship. If $V = \overline{V}$, VCs back only Es with $w \in [C - \bar{p},\ w^*]$. However, the inflow of free VCs is

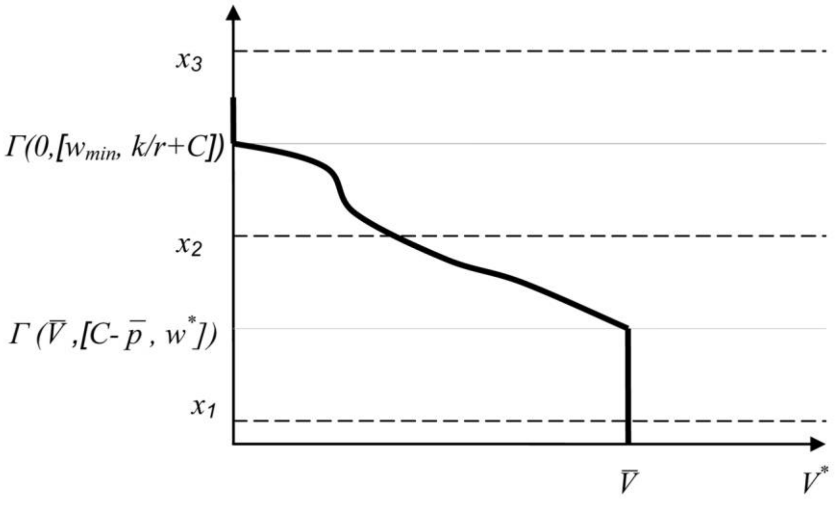

FIG. 4.—Market clearing

smaller than $\psi([w^*,\ C - \bar{p}])$, and hence, we are unable to determine which of these Es are supported in equilibrium. Since all of the surplus goes to the VCs, these Es are all indifferent between receiving VC financing and abandoning their projects. In either case, the payoffs of all agents are the same across all of the equilibria.

B. *The Selection Mechanism*

Note that the quality of a project does not directly depend on whether the VC is present. Rather, differences between the outcomes of VC-backed versus solo ventures stem from differences in financing decisions alone, that is, from agents' equilibrium behavior. Next, we compare projects run by rich solo Es with those financed by VCs. Then we investigate how the wealth of a solo E affects her decisions.

Since the VC can always move on to a new company, looking after a nonperforming company entails a forgone-earnings cost that an E does not face. The impatient VC imposes a higher quality hurdle than a rich solo E does. In addition, the opportunity cost of the VCs induces earlier termination times of VC-backed firms than those of rich Es.

PROPOSITION 1. Suppose that $V^* > 0$. Then for all equilibrium sharing rules s^*, (i) $\pi^S_{\min} < \pi^{VC}_{\min}(V^*, s^*)$ and (ii) $T^{VC}(\pi, s^*, V^*) < T^S(\pi)$ whenever $T^S(\pi) > 0$.

This proposition has a strong implication for the expected profits of successful projects financed by VCs and rich solo Es. At any point in time, the threshold of a VC above which he is willing to support a project is higher than that of a rich solo E. As a consequence, the expected profit of a VC-backed project conditional on success is higher than that of a rich E. Remark 1 states this formally.

REMARK 1. If $V^* > 0$, then for all equilibrium sharing rules s^* and for all $\tau \in \mathbb{R}_+$,

$$E_\pi(\pi|\tau < T^S(\pi)) < E_\pi(\pi|\tau < T^{VC}(\pi, s^*, V^*)).$$

As we mentioned in the introduction, this inequality is consistent with data; successful VC-backed companies appear to be more profitable than others. We stress that, in our model, this arises solely from the termination decisions of the VCs due to their opportunity cost, V^*, and not directly from their somehow improving the management or the marketing of the companies. Our selection mechanism also differs from a selection story in which a VC can select better projects in the first place. Our VCs have no superior information about π before they commit to financing a project.

We turn our attention to the effect of the wealth of solo Es. A poor E who would run out of money before the optimal termination time is also more selective; knowing that she may run out of money, a poor E requires her project to be of sufficiently high quality in order to continue investing. Indeed, we can show that she generally will terminate her project earlier for any π.

PROPOSITION 2. (i) $\pi^S_{\min}(w)$ is decreasing in w and constant on $[C + k/r, \infty)$. (ii) $T^S_w(\pi)$ is increasing in w and constant on $[C + k/r, \infty)$.

The net effect on terminations and expected profit conditional on success is thus ambiguous and depends on the distribution of wealth among the solo Es.

C. *Welfare Analysis*

We now show that the competitive equilibrium is efficient if either the measure of VCs is small or it is large and a VC can profit from contracting with an E even if she has zero wealth.

THEOREM 2 (The welfare theorem). The competitive equilibrium is socially optimal if either (i) $x \geq \bar{x}$ and $w_{\min} = 0$ or (ii) $x \leq \underline{x}$.

Social efficiency.—We next characterize the socially efficient allocations. Note that solo Es internalize both the benefits and the costs of their projects: Their decisions regarding their projects are socially optimal. Also note that socially optimal termination of a VC-backed project does not depend on E's wealth. Therefore, the efficient allocation can be described by the pair $(A^*, T^*(\cdot))$, where A^* is the set of VC-backed Es and $T^*(\pi)$ is the termination time of a VC-backed project with payoff π.

If it is optimal to VC-back an E who would otherwise go solo, then efficiency requires that every poorer E who is constrained should also be VC-backed. Formally,

$$\sup A^* > w^* \Rightarrow \{w | 0 \leq w \leq \min\{\sup A^*, C + k/r\}\} \subset A^*. \tag{15}$$

We next establish a relationship between A^* and $T^*(\cdot)$. To this end, we first consider an efficient allocation and compute the social value generated by one additional VC. By (15), this marginal VC can be assumed to back the E with wealth $\tilde{w} = \sup A^*$. Let $\tilde{q}$ denote the expected surplus generated by this E in the absence of VCs, that is, $\tilde{q} = \max\{0, -C + Q^S(\tilde{w})\}$. The social value added by the marginal VC, W, is determined by the following Bellman equation:

$$W = -(C + \tilde{q}) + \max_{T(\pi), \pi \in \mathbb{R}_+} \int \int_0^{T(\pi)} \left[\pi + W - \frac{a + k}{h(t)}\right] e^{-rt} dF(t) + We^{-rT(\pi)}(1 - F[T(\pi)]) dG(\pi). \tag{16}$$

The term $C + \tilde{q}$ is the total initial social cost of the project backed by the marginal VC, C is the physical cost, and $\tilde{q}$ is the opportunity cost. The rest of the expression corresponds to maximizing social surplus by choosing the optimal termination time given the continuation value W. Equation (16) has a unique solution for W. The reason is that the right-hand side is (weakly) positive at $W = 0$ since the project generates at least $\tilde{q}$ and is strictly smaller than W if W is large since $C + \tilde{q} > 0$.[13] In addition, its slope is strictly smaller than one by the envelope theorem. The existence of a unique solution then follows from the intermediate value theorem. The corresponding optimal termination time is defined by

$$T^*(\pi, W) = \begin{cases} h^{-1}\left(\dfrac{a + k + rW}{\pi}\right) & \text{if } \pi > \pi_{\min}(W) \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

where $\pi_{\min}(W)$ denotes the cutoff below which it is optimal to terminate immediately.

The termination decisions determine the average duration of a VC-backed project $t^*(W) = E_\pi[T^*(\pi, W)]$. As in (12), the following condition guarantees that the inflow and outflow of VCs are equal:

$$d_\Psi(A^*) \begin{cases} \leq \dfrac{x}{t(W)} & \text{if } W = 0 \\ = \dfrac{x}{t(W)} & \text{if } W \geq 0. \end{cases} \tag{18}$$

To summarize, the pair $(A^*, T^*(\cdot))$ defines a socially efficient allocation if (i) (15) holds, (ii) $T^*(\pi) = T^*(\pi, W)$, and (iii) (18) is satisfied with W defined by (16). The argument showing that these conditions are also sufficient and that the efficient allocation is essentially unique is similar to the proof of theorem 1, and hence, it is omitted.

The proof of the welfare theorem.—We show that if either $x \leq \underline{x}$ or $x \geq \bar{x}$ and $w_{\min} = 0$, the equilibrium set of VC-backed Es, A, and termination times define an efficient allocation.

Case i: $x \geq \bar{x}$ and $w_{\min} = 0$. Theorem 1 and lemma 6 imply that $V^* = 0$ and $[0, k/r + C] \subset A$. In addition, $w_{\min} = 0$ and lemma 3 imply that each equilibrium contract specifies $s = a/(a + k)$, and hence, the termination decisions are $T(\cdot, a/(a + k), 0)$ defined by (7). We show that the pair $(A, T(\cdot, a/(a + k), 0))$ defines an efficient allocation.

Since every financially constrained E is VC-backed in equilibrium, (15) is satisfied. The marginal VC finances an E who is financially uncon-

[13] As W increases, it is optimal to terminate more projects at once. Hence, the right-hand side $-W$ converges to $-(C + \tilde{q})$.

strained, so the surplus generated by the poorest VC-backed E is $\tilde{q} = Q^S(k/r + C) - C$. Therefore, (16) implies $W = 0$ and can be written as

$$Q^S(k/r + C) = \max_{T(\cdot)} \int\int_0^{T(\pi)} \left[\pi + \frac{a+k}{h(t)}\right] e^{-rt} dF(t) + dG(\pi),$$

which is just the surplus of an unconstrained solo E. In particular, the efficient termination time (17) with $W = 0$ is just the unconstrained E's termination time as defined by (2). Therefore, by (2) and (7), $T(\pi, a/(a+k), 0) = T^*(\pi, 0)$. Finally, note that (12) and (18) coincide.

Case ii: $x \leq \underline{x}$. Theorem 1 and lemma 6 imply that $V^* = \overline{V}$ and $A \subset [w^*, k/r + C]$. The equilibrium contract in this case is $(\bar{s}, \bar{p})$ defined by lemma 4. We show that the pair $(A, T(\cdot, \bar{s}, \overline{V}))$ determines a socially optimal outcome.

Since only those Es who would not otherwise go solo are VC-backed, $\sup A \leq w^*$ and, hence, A satisfies (15). In this case, the VCs capture all the surplus generated by VC-backed projects. Therefore, the VC's equilibrium value can be written as follows:

$$\begin{aligned}\overline{V} = -C + \max_{T(\pi), \pi \in \mathbb{R}_+} \int\int_0^{T(\pi)} \left[\pi + \overline{V} - \frac{a+k}{h(t)}\right] e^{-rt} dF(t) \\ + \overline{V} e^{-rT(\pi)}(1 - F[T(\pi)]) dG(\pi).\end{aligned}$$

The marginal VC finances an E who would otherwise invest his wealth in a bank, so $\tilde{q} = 0$. This implies that the previous equation is simply (16). Therefore, $W = \overline{V}$ and $T(\pi, \bar{s}, \overline{V}) = T^*(\pi, \overline{V})$. Finally, (12) and (18) are identical.

D. *Discussion of the Welfare Theorem*

We assumed throughout that π is the social value of a project. However, our analysis of competitive equilibria goes through even if π does not equal the social value. For example, implementing more projects might result in technological spillovers, which, in turn, can generate economic growth. We return to this issue in Section V.B. In the presence of such externalities, the welfare and policy implications of our model are different. However, even if the social and private values of a project differ, theorem 2 still justifies the use of simple equity contracts for cases i and ii of this theorem. In case i, Es capture all the surplus from the projects and $V^* = 0$. Therefore, any contract that would generate a strictly positive payoff to a VC would be rejected by Es. In particular, a VC cannot deviate profitably by offering a more complicated contract even if such a contract is available. In case ii, the VC captures all the surplus from the projects and Es break even. Again, the VC has no incentive to adopt a different contract.

The validity of the first welfare theorem might seem surprising in our setting for two reasons. First, agency problems could arise because neither the VC's investment nor the E's effort is contractible. Second, agents are not price takers: VCs are strategic when offering contracts. We next explain why, despite these issues, the competitive equilibrium is efficient in the environments described in theorem 2. In particular, we argue that the contractibility of the sharing rule eliminates the agency problem and that the VCs are able to internalize their social value.

Intuition for the equilibrium contract.—The VC and E must both want to support the project until the socially optimal termination time and no longer. Recall that, in the environment of theorem 2, $V^* = W$, and the equilibrium sharing rule is given by $s = a/(a + k + rW)$. We argue that the sharing rule s provides the right incentives for both agents because the marginal cost of a VC-backed project is shared correctly between the VC and E. The VC invests k and incurs an opportunity cost of rW, while E exerts effort a. Then E cares only about her own cost, a, and not the social cost $a + k + rW$. However, if $s = a/(a + k + rW)$, then E's benefit is $[a/(a + k + rW)]\pi$ instead of the social benefit π. This means that the objective function of E is equal to the objective function of the social planner scaled down by the constant $a/(a + k + rW)$. Similarly, the objective function of the VC is scaled down by $(k + rW)/ (a + k + rW)$. Scaling does not actually affect decisions, so the VC and E both choose the same T that the planner would. Therefore, our model provides an explanation for why we observe equity contracts between VCs and Es, and not, for example, debt or labor contracts.

The VC's market value.—We have shown that when the competitive outcome is efficient, the social value of a VC coincides with his market value: $V^* = W$. In general, if $x \in (\underline{x}, \bar{x})$, the social value of the marginal VC exceeds his market value. A VC could increase the social surplus by financing those Es who invest with a bank. Unfortunately, the VCs cannot extract the full social surplus from poor Es because they are unable to compensate the VCs up front for long-lasting project support. If it were possible to write more complicated contracts, the equilibrium allocation would be efficient for a larger set of parameters. In particular, the VCs would be able to extract sufficient surplus from liquidity-constrained Es if the sharing rule s could increase over time. Recall that we have restricted attention to time-independent sharing rules, and hence the E is indifferent between exerting effort and shirking at the time of termination but strictly prefers to exert effort at every earlier point in time. If s were permitted to vary over time, it would be possible to make E indifferent between working and shirking prior to project termination, and such contracts would allow surplus to be extracted from poorer Es without violating incentive constraints.

V. Fitting Data

We now fit the model to data on VC-backed projects. We have two goals. The first is to learn what features of the data the model explains; the second is to infer the returns to VCs and to solo Es.

In this section, we assume that case i of theorem 1 holds, that is, that $x \leq \underline{x}$, so that the VC is in short supply and so that his payoff is $\overline{V}$, as shown in figure 3.[14] This simplifies things greatly because the decisions regarding VC-backed projects then do not depend on Es' wealth distribution. Moreover, the assumption on the scarcity of the VCs is probably appropriate for the vintages 1989–93 of funds that our data cover. On these vintages, VCs as well as the LPs of private equity funds received above-average returns, as table 3 below shows. Online Appendix E argues that if the parameter x was estimated as part of a larger model, the constraint $x \leq \underline{x}$ would be satisfied.

Next, we define precisely those objects of our model that we intend to fit to the corresponding features of the data. After that, we describe the data and then discuss the simulations.

The distribution of successful-exit payoffs, $\Gamma_t(\pi)$.—Let $\hat{\tau} > \tau_m$ denote the age at which $h(t) = (a + k + rV)/\pi^{VC}_{\min}$. For $t \in (0, \hat{\tau})$, no project is terminated, and successes come from the same truncated distribution $G(\pi | \pi \geq \pi^{VC}_{\min})$. At $t = \hat{\tau}$ the truncation point for π, namely $(a + k + rV)/h(t)$, starts to rise. The distribution of π among projects that bear fruit at date t is

$$\Gamma_t(\pi) \equiv \begin{cases} G(\pi | \pi \geq \pi^{VC}_{\min}) & \text{for } t < \hat{\tau} \\ G\left(\pi | \pi \geq \dfrac{a + k + rV}{h(t)}\right) & \text{for } t \geq \hat{\tau}. \end{cases} \tag{19}$$

The probability, $\Phi(t)$, *that a nonperforming project is terminated by age t.*—Conditional on no success, a project is terminated before age t if π falls below the VC's reservation quality before t. Define

$$\Phi(t) \equiv \Pr(T \leq t | \tau \geq t). \tag{20}$$

[14] The notation for this case was $\overline{V}$, $\bar{s}$, $\bar{p}$, etc., but in this section we write V, s, and p instead. Variables that pertain to VC-backed projects and those that pertain to solo projects have the superscripts VC and S as in, e.g., $\pi^{VC}_{\min}$ and $\pi^{S}_{\min}$. By $\pi^{VC}_{\min}$ we mean $\pi^{VC}_{\min}(\overline{V}, \bar{s})$ throughout Sec. V.

Lemma 2 states that project π is terminated either at once or at date $T \geq \tau_m$ satisfying (8). Thus $\Phi(t)$ jumps immediately from zero to $G(\pi_{\min}^{VC})$, remains there until $t = \hat{\tau} \equiv h^{-1}((a + k + rV)/\pi_{\min}^{VC})$, and then rises again.

The survivor function, $S(t)$.—A project "survives" through date t if it has neither succeeded $(\tau > t)$ nor terminated $(T(\pi) > t)$. Since τ and π are independent, the fraction surviving is

$$S(t) = (1 - F[t])(1 - \Phi[t]) = \exp\left\{-\int_0^t [h(s) + \psi(s)]\,ds\right\}, \tag{21}$$

where

$$\psi(t) = \frac{\Phi'(t)}{1 - \Phi(t)}$$

is the termination hazard. Thus h and ψ are competing hazards: $-S'(t)/S(t) = h(t) + \psi(t)$.

The J curve, $J(t)$.—The term is often used to describe the cumulative net income of a venture fund as it ages, according to Wikipedia. For us, in contrast, the J curve is the expected cumulative net revenue of a representative company. If all the portfolio firms of a fund were to start at the inception of the fund, the two concepts would be the same. Expected investment at age t is $kS(t)$, and successes occur at the rate $f(t) = h(t)S(t)$. Then J solves the ordinary differential equation (ODE)

$$J'(t) = \left[-k + h(t)(1 - s)\int \pi\, d\Gamma_t(\pi)\right] S(t), \tag{22}$$

with the initial condition $J(0) = -p$.

The VC's excess return α^{VC}.—We define the excess return of the VC, α^{VC}, as the difference between the internal rate of return (IRR) on the flow of expected profits and the rate of discount r. Thus α^{VC} is the solution for α to the equation

$$0 = -p + \int_0^{12} e^{-(r+\alpha)t} J'(t)\,dt. \tag{23}$$

The funds our data cover do not extend beyond year 12; if the VC receives net revenues after the fund closes, then α^{VC} is an underestimate but not a large underestimate because the years 13 and beyond are heavily discounted.

The solo E's excess return $\alpha^S(w)$.—The model allows us to calculate the returns of solo Es by estimating the parameters exclusively from the data on VC-backed projects. The solo E's excess return, $\alpha^S(w)$, depends on E's wealth, w. It is calculated in the same way as α^{VC}, but with the following changes in (22): (i) a solo E must pay C up front instead of just p, (ii) in the event of success she receives the entire π, and (iii) she may run out of money, which effectively lowers $h(t)S(t)$. See equations (C8)–(C12) in online Appendix C for details.

Parameterization.—We use the following functional forms for G and F:

$$G(\pi) = 1 - \left(\frac{\pi}{\pi_0}\right)^{-\lambda}, \quad \pi \geq \pi_0, \ \lambda > 1, \tag{24}$$

$$F(\tau) = \frac{\rho}{2+\rho}\left[\frac{\min(\tau, \tau_m)}{\tau_m}\right]^2 + \frac{2}{2+\rho}\left\{1 - \left[\frac{\max(\tau, \tau_m)}{\tau_m}\right]^{-\rho}\right\}. \tag{25}$$

Thus π has a Pareto distribution with mean $\lambda/(\lambda - 1)\pi_0$. The hazard $h(\tau)$ rises up to a peak at τ_m, beyond which it declines. Then (C7), (24), and (25) imply the following proposition.

Proposition 3. For $t \geq \hat{\tau}$, (i) $h(t) = \rho/t$, (ii) $\psi(t) = \lambda/t$, and (iii)

$$\Pr(\text{succ}|\text{exit}) = \frac{h(t)}{h(t) + \psi(t)} = \frac{\rho}{\rho + \lambda}. \tag{26}$$

Data.—The data come from the VentureXpert database provided by Thompson Venture Economics, covering VC investments in 1,355 US companies. VC investments in these companies start between 1989 and 1993 and extend to 2001, when our sample ends. Guler (2003, 2007) and Jovanovic and Szentes (2007) analyzed this sample; from it we drop 403 firms with incoherent information, reducing the total number to 952. Of these, 27 percent had an IPO and 17 percent were sold privately. The remaining 56 percent generated no revenue; for them we assumed that a firm was "terminated" a year after its last recorded investment. Panels 1–6 of figure 5 plot the data. Depending on their vintage, we lose coverage of firms with ages between 9 and 12. Appendix B details the procedures we followed.

Before we proceed, one feature of the model needs to be changed. Because a perfect signal on π arrives at once, the VC terminates immediately all projects for which $\pi \leq \pi^{VC}_{\min}$ and then does not terminate any more until date $\hat{\tau} > \tau_m$. Then (21) states that S exhibits a downward jump at $t = 0$, and (19) states that $\Gamma_t(\pi)$ is fixed for $t \in (0, \hat{\tau})$, so its mean is $E(\pi|\text{VC success})$. Neither property is in the data, and we shall therefore assume that learning π takes time.

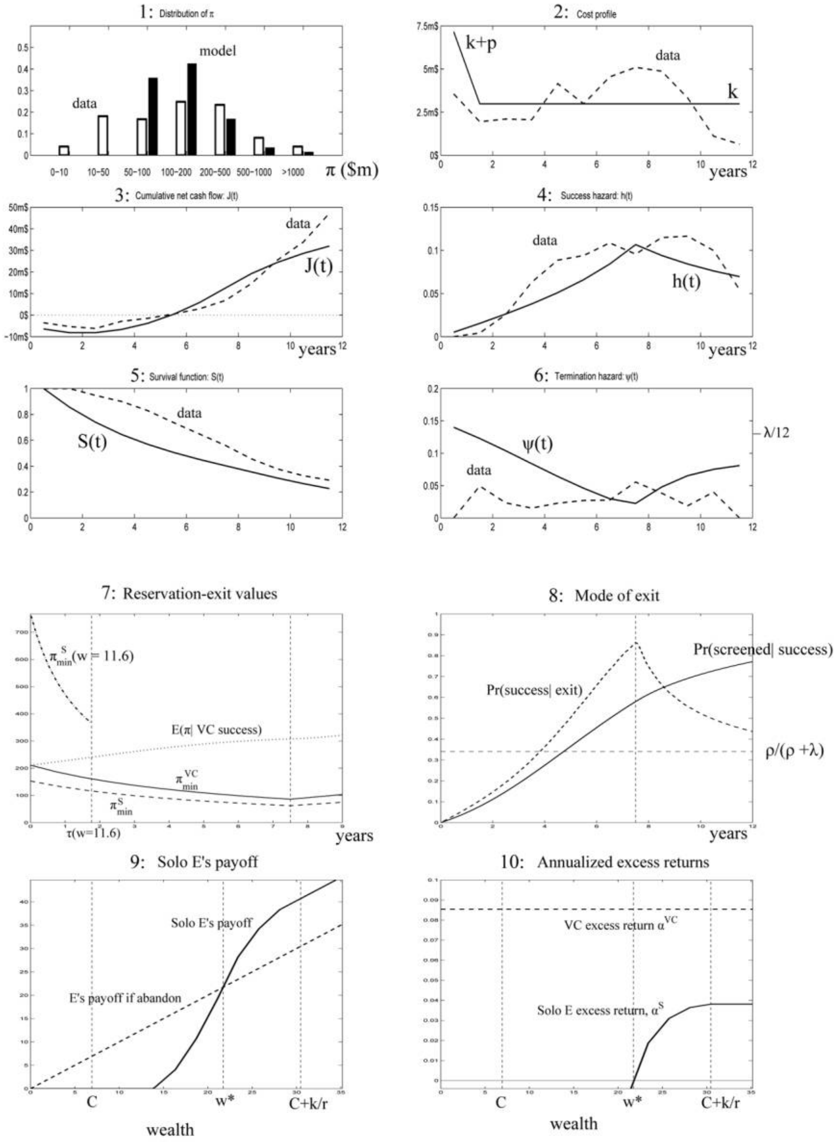

FIG. 5.—Simulation 1

Introduction of a delayed signal on π.—We assume that there is a random delay for both parties to simultaneously learn π. As a result, every project is now supported for a positive length of time, T_0, if no signal arrives. If the signal arrives at date $t < T_0$, the reservation qualities are denoted by $\pi^{VC}_{\min}(t)$ and $\pi^{S}_{\min}(t)$ for VC-backed and solo projects. The flow of terminations is now positive for all $t \leq T_0$. This extension does not affect our theoretical results, and our welfare theorem still holds. The characterization of the optimal termination decisions is in Appendix A, where we define $B(t)$ to be the probability that π is learned before t. For the simulations we assume that

$$B(t) = 1 - e^{-\delta t}, \quad t \geq 0. \tag{27}$$

Parameter choices.—The rate of discount, r, was chosen on the basis of evidence on the required real rate of return on investments with characteristics similar to those in our sample. We use the standard asset-pricing relation

$$r \equiv r_f + \beta_{VC}(r_m - r_f). \tag{28}$$

The risk-free rate $r_f = 0.026$ was set at the mean annualized real return on 3-month Treasury bills from 1980 to 1999, which is the sample period for Jones and Rhodes-Kropf's (2004) data on returns of VC and buyout funds. The market return $r_m = 0.082$ was set at the mean of the return on a value-weighted portfolio of stocks listed on the New York Stock Exchange, the American Stock Exchange, and NASDAQ provided by the Center for Research in Security Prices over the same period. The value $\beta_{VC} = 1.80$ is the covariance with systematic market risk calculated as the sum of the five quarterly β coefficients in Jones and Rhodes-Kropf's table 1, panel B. Equation (28) then yields $r = 0.127$, the predicted cost of funds for both the VC and the solo E.[15]

The remaining eight parameters ($\pi_0, C, k, a, \lambda, \delta, \rho$, and τ_m) were chosen to fit the data plotted in the first six panels of figure 5, namely, the distributions of revenues and costs, the success and termination hazards as a function of age, and the relation between a firm's age and its cumulative net revenue flow. We did three simulations. The baseline, Sim 1, produced the best overall fit with the parameters, shown in the top row of table 1, and the results are plotted in figure 5. Sim 2 raises λ, making thinner the right tail of the distribution of π's and lowering their mean, but compensates for this by raising the speed of detection, δ, and lowering the up-front cost C. Sim 3 uses parameters that lead to a lower α^{VC}. Appendix D reports plots and further discussion of the simulations.

The last seven columns of table 1 list estimates of endogenous variables: α^{VC} and $\alpha^S(w)$ were defined above; $\alpha^{\text{rich}} \equiv \lim_{w\to\infty} \alpha^S(w)$ is the predicted excess return of a rich solo E; V is the lifetime value of a VC; $\sigma \equiv Q^S - C$ is the lifetime value of a project to a rich solo E (see fig. 2); s is E's equity share in the contract; p is the VC's up-front payment; and

[15] Hall and Woodward (2007) find $\beta_{VC} = 1.7$, which would imply that $r = 0.121$.

TABLE 1
Parameters and Outcomes

	Parameter Values									Outcomes						
Simulation	r	π_0	C	k	a	λ	δ	ρ	τ_m	α^{VC} (%)	α^{rich} (%)	V	σ	s	p	T_0
Sim 1	12.7	75	6.9	3	3.6	1.55	.2	.8	7.5	8.6	3.4	20	8.5	.40	4.8	15.1
Sim 2			3.6			1.73	.4			7.7	3.1	16	5.4	.42	2.4	13.9
Sim 3			12.6			1.6	.3			1.8	.7	5	1.0	.50	7.3	14.4

T_0 is the age at which the VC terminates a project on which no signal has come in. How should V relate to σ? A rough guess would go as follows: if a VC could dispose of each company at around the modal success age $\tau_m = 7.5$ and if he followed the same termination policy as the rich solo E, the VC would have a lifetime value of $\sigma/(1-e^{-r\tau_m}) = 1.63\sigma$. In fact, the VC can (and does) do better than this in that the ratio V/σ ranges between 2.4 in Sim 1 and 5 in Sim 3.

A. *Model Fit*

In figure 5 the solid lines depict the expected values of the variables that the model generates for Sim 1.

Panel 1: The distribution of π.—Panel 1 plots the distribution of exit values (the counterpart of the π's) measured in millions of 2001 dollars. They are observed only for successes. The solid distribution is a weighted average of the distributions $\Gamma_t(\pi)$ given in (19) for the case $\delta = \infty$, but the plot is for $\delta = 0.2$.[16] Then each Γ_t is a weighted average of screened and unscreened projects; see equation (C1). The model understates the predicted dispersion of π, but this is easily remedied if instead of a perfect signal the parties were to receive an unbiased noisy signal of π. The distribution of realized π's would then be a mean-preserving spread of the solid distribution. Since the VC and E are risk neutral, their optimal termination and effort decisions would not change and, as a result, neither would panels 2–10.

Panel 2: Investment-cost profile.—The VC invests k in every period except the first, when he invests $k + p$, where $p = (1 - s)C$ by lemma 4. The value $s = 0.40$ matches Kaplan and Strömberg's (2003, table 2) evidence; they report that pooled over rounds, the claim of founders is 31.1 percent, of VCs 46.7 percent, and of non-VC investors 22.2 percent, which suggests $s \approx 31.1/(31.1 + 46.7) = 0.40$. These estimates imply that the VC finances 60 percent of C and receives 60 percent of the equity.

[16] The formulas for the case $\delta < \infty$ are in App. C.

TABLE 2
EQUATION (26) (ii) AND (iii) EVALUATED AT $t = 12$

	$\lambda = 1.55$ SIM 1		$\lambda = 1.73$ SIM 2		$\lambda = 1.6$ SIM 3	
	$\delta = \infty$	$\delta = .2$	$\delta = \infty$	$\delta = .4$	$\delta = \infty$	$\delta = .3$
$\psi(12) = \lambda/12$	.13	.08	.14	.06	.13	.07
$P(\text{succ}\vert\text{exit}) = \rho/(\rho + \lambda)$	.34	.43	.32	.52	.33	.49

Panel 3: The J curve.—The intercepts of the curves in panels 2 and 3 both depend on p. Sim 1 fits the J curve well, except in periods 10–12, which are heavily discounted. Sim 2 and especially Sim 3 underpredict the J curve and imply values of α^{VC} far below what one would expect on the basis of the private-equity returns that we report in table 3 below.

Panels 4, 6, and 8: The hazards.—Panel 4 plots the success hazard $h(t)$. Panels 6 and 8 plot the simulated finite-δ counterparts of the expressions in parts ii and iii of proposition 3. Table 2 compares the finite-δ simulations to the $\delta = \infty$ case that the proposition covers. Panels 6 and 8 indicate on their vertical right axes the $\delta = \infty$ values of $\psi(12)$ and Pr(succ|exit), respectively. As t grows, all three simulations show a slow convergence of $\psi(t)$ and Pr(succ|exit) toward these values. Since δ is twice as high in Sim 2, Pr(succ|exit) is larger.

Panel 5: Survivorship.—The dashed line plots the number that survived up to the beginning of year t. The predicted S dips below the actual survivals immediately. As panel 6 also shows, the model implies too many early terminations early on: The signal on π arrives too early, leading to an early removal of the low-π projects. The fit would improve if, instead of δ being constant, it had an inverted-U shape, similar to that of $h(t)$.[17] The data also show fewer terminations at the high ages than the model generates, but of the 28 percent of companies we infer are "alive" at age 12, many may in fact be defunct (Ruhnka, Feldman, and Dean 1992). Indeed, panel 2 shows investment dropping off sharply in companies that are 10 years and older, suggesting that undetected terminations have taken place by then.

Panel 7: Project selectivity of VCs and Es.—Since h is inverted-V-shaped, $\pi^{VC}_{\min}(t)$ and $\pi^{S}_{\min}(t)$ (the reservation quality of the rich solo Es who receive information at t) are both V-shaped. The curve $E(\pi|\text{VC success at } t)$ starts from its unconditional mean $\pi_0\lambda/(\lambda - 1)$ and then rises as low π's are gradually terminated. The differential selectivity of VCs and Es raises

[17] The data-trimming procedure described in App. B eliminated $361/1{,}355 = 27$ percent of the original firms because they reported no cash flows—negative or positive. Had we treated these as terminations instead of data errors, we would have obtained a large initial termination spike for the dashed line in panel 6, resulting in a much better fit for ψ as well as for $S(t)$ in panel 5, where the dashed line would show a sharp initial decline.

TABLE 3
Real IRR to Limited Partners by Fund Vintage

	Vintage Years			
	1986–88	1989–93	1994–2000	2001–8
IRR	11.7	22.4	12.0	13.6

the IPO values and termination rates of VC-backed firms above those of rich solo Es and explains why VC-backed companies are worth more at IPO (Megginson and Weiss 1991; Hochberg 2012). However, the implications are reversed for poor Es who run out of money before τ_m (when the chances of success are highest) and who, as a result, are more selective. Panel 7 considers a solo E with $w = 11.6$ who, according to (1), would run out of money at $t = 1.7$ and shows her reservation π to be far above $\pi^{VC}_{\min}$. The poor E is extremely selective because she must gamble on an early success, which, given the shape of $h(t)$, is an unlikely event. Equations (C3), (C4), and (C8) show how $\pi^{VC}_{\min}(t)$ and $\pi^{S}_{\min}(w)$ are calculated.

Panel 9: Es' occupation choice.—This panel is the simulated version of figure 2. The net value of going solo is the difference between the curve and the 45-degree line in panel 9 and in figure 2 above. See equations (C10)–(C12) for details on $Q^S(w)$. The critical wealth level $C + k/r$ is the minimal w a solo E needs to support the project forever if she wishes to.

Panel 10: Excess returns of VCs and solo Es.—Panel 10 shows that $\alpha^S(w)$ is substantially below α^{VC} but positive and rising to 3.5 percent. The estimate $\alpha^{VC} = 8.6$ percent may seem high, yet it is less than what the fund-level returns data imply as we shall now show. Since a VC supplies both his skill and his money, $r + \alpha^{VC}$ is comparable to combined returns to all the partners of a venture fund. Our data are on portfolio firms in funds of vintage 1989–93; a summary of returns to LPs of private equity is in table 3.[18] The IRR in the table is the return that the LPs receive after the GPs have taken roughly 2 percent overhead and 20 percent of any profits on the LPs' investment. Hence the reported IRR is conceptually at least 2 percent below what we call $r + \alpha^{VC}$. Our estimate of $r + \alpha^{VC}$, $12.7 + 8.6 = 21.3$ percent, is 3 percentage points lower than the returns one would expect to find on the basis of the 22.4 percent return that accrued to the LPs of the funds of 1989–93 vintage. A similar pattern for IRRs by vintage is reported in table 2 of Hochberg et al. (forthcoming); their IRRs are lower and more in line with our estimates of $r + \alpha^{VC}$.

[18] Source: Cambridge Associates (2011, 7, col. 2). We pooled over the four sets of years as indicated in table 3 and then corrected for inflation, subtracting the annualized 10-year growth of the consumer price index starting from each vintage year.

The returns are vintage specific: The stock market was unusually high in the late 1990s, just as these projects were maturing, and the table indicates that lower α's obtained for other vintages of companies. For other vintages of funds, scarcity of projects and not VCs may be the appropriate assumption. In the extreme case in which VCs are in infinitely elastic supply, E would get all the rents and $\alpha^{VC} = 0$. Appendix E discusses in more detail the consequences of relaxing the VC-scarcity assumption.

The estimate $0 \le \alpha^S(w) \le 3.5$ percent is high compared to negative α's reported by Moskowitz and Vissing-Jorgensen (2002) for the self-employed and by Astebro (2003) for independent inventors. Vereshchagina and Hopenhayn (2009) argue, however, that estimated returns rise when one factors in the options entrepreneurs can turn to if they fail.

B. *Discussion of the Results*

The role of the VC.—In the model, a VC does not raise value directly as an input in production or management. Rather, he acts as lender as well as overseer and screener of projects. Thus V is the private lifetime value per venture company derived by the VCs and the LPs of a fund combined. Since a VC supplies both his skill and his money, a "limited supply of VCs" could reflect a shortage of human capital or a shortage of liquidity. Since venture funds are usually oversubscribed, the binding constraint may indeed be VCs' human capital, and not liquidity.

Private versus social returns.—Our estimates of α^{VC} and α^S are excess private returns; they are based on the π's, which are private values. A private value can exceed the social value if the project takes rents away from other projects, and it can be less than the social value if the project bestows knowledge externalities on others.

Scale effects.—We have omitted explanatory variables that could help improve the fit. For example, companies in the pharmaceutical industry make larger investments (a, k, C), they face longer delays (higher τ_m), and they receive higher payoffs π. Moreover, the form of the optimal contracts would also depend on all ex ante available information about the project's fundamentals. On the other hand, such differences create a sorting motive: A poor E could choose a project with low C, thereby reducing ex post inefficiency, because to compensate the VC, she might not then need to settle for an inefficiently low equity share s.

Aggregate shocks and expectations.—Aggregate exit values fluctuate, as table 3 shows, and the changes are not perfectly foreseen. Our methodology assumes away aggregate risk and time effects: The joint population distribution of the pair (π, τ) realized ex post is the same that agents expected ex ante. Since many of our companies matured during the stock market boom, we have implicitly assumed that VCs and Es alike expected the boom to occur. If the boom was in fact a surprise, more terminations would have been optimal for low ages than our model generates.

Other functional forms.—The Pareto form of G in (24) leads to analytic simplification such as equation (26), but its thick left tail causes too many terminations. A more flexibly parameterized $G(\pi)$ and $k(t)$ would fit better. Also, "lemons ripen faster than cherries" (Lerner 1998); in other words, bad projects are perhaps revealed earlier than good projects, so that π and the signal-arrival age may be negatively correlated.

VI. Conclusion

We modeled the market for venture capital in which VCs play a lending and monitoring role and in which they do not directly add value to projects they finance. For a wide range of distributions of project quality and wealth on the one hand and for a wide range of VC supply on the other, the model says that contracts should entail a fixed sharing rule and an up-front payment.

Also, we fit an equilibrium model to the data, a rare event in the corporate finance literature. The exercise is quantitative: it goes beyond a qualitative test of a model's comparative statics properties by means of regressions. We showed how the returns to venture capital and VC termination behavior depend on the distribution of project qualities, on the distribution of waiting times for the projects to mature, and on the scarcity of VCs.

References

Astebro, Thomas. 2003. "The Return to Independent Invention: Evidence of Unrealistic Optimism, Risk Seeking or Skewness Loving?" *Econ. J.* 113 (January): 226–39.

Basaluzzo, Gabriel. 2006. "Entrepreneurial Teams in Financially Constrained Economies." Manuscript (February), Instituto Tecnológico Autónomo de México.

Bergemann, Dirk, and Ulrich Hege. 2005. "The Financing of Innovation: Learning and Stopping." *RAND J. Econ.* 36 (Winter): 719–52.

Buera, Francisco. 2009. "A Dynamic Model of Entrepreneurship with Borrowing Constraints: Theory and Evidence." *Ann. Finance* 5 (3–4): 443–64.

Cambridge Associates. 2011. "U.S. Private Equity Index and Selected Benchmark Statistics." Report (June), Cambridge Assoc.

Cochrane, John. 2005. "The Risk and Return of Venture Capital." *J. Financial Econ.* 75 (January): 3–52.

Guler, Isin. 2003. "An Empirical Examination of Aggregate Patterns in U.S. Venture-Capital Decision Making." Manuscript (March), Wharton School, Univ. Pennsylvania.

———. 2007. "Throwing Good Money after Bad? Political and Institutional Influences on Sequential Decision Making in the Venture Capital Industry." *Administrative Sci. Q.* 52 (June): 248–85.

Hall, Robert, and Susan Woodward. 2007. "The Incentives to Start New Companies: Evidence from Venture Capital." Working Paper no. 13056 (April), NBER, Cambridge, MA.

Hochberg, Yael. 2012. "Venture Capital and Corporate Governance in the Newly Public Firm." *Rev. Finance* 16 (2): 429–80.

Hochberg, Yael, Alexander Ljungqvist, and Annette Vissing-Jorgensen. Forthcoming. "Informational Hold-up and Performance Persistence in Venture Capital." *Rev. Financial Studies*.

Holmes, Thomas, and James Schmitz. 1990. "A Theory of Entrepreneurship and Its Application to the Study of Business Transfers." *J.P.E.* 98 (April): 265–94.

Inderst, Roman, and Holger Müller. 2004. "The Effect of Capital-Market Characteristics on the Value of Start-up Firms." *J. Financial Econ.* 72 (May): 319–56.

Jones, Charles M., and Matthew Rhodes-Kropf. 2004. "The Price of Diversifiable Risk in Venture Capital and Private Equity." Manscript (July), Grad. School Bus., Columbia Univ.

Jovanovic, Boyan, and Balázs Szentes. 2007. "On the Return to Venture Capital." Working Paper no. 12874 (January; rev. September), NBER, Cambridge, MA.

Kaplan, Steven, and Antoinette Schoar. 2005. "Private Equity Performance: Returns, Persistence and Capital Flows." *J. Finance* 60 (August): 1791–1823.

Kaplan, Steven, and Per Strömberg. 2003. "Financial Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts." *Rev. Econ. Studies* 70 (April): 281–316.

Lerner, Joshua. 1998. "Comment on Bergemann and Hege." *J. Banking and Finance* 22:736–40.

Ljungqvist, Alexander, and Matthew Richardson. 2003. "The Cash Flow, Return and Risk Characteristics of Private Equity." Working Paper no. 9454 (January), NBER, Cambridge, MA.

Megginson, William, and Kathleen Weiss. 1991. "Venture Capitalist Certification in Initial Public Offerings." *J. Finance* 46 (July): 879–903.

Michelacci, Claudio, and Javier Suarez. 2004. "Business Creation and the Stock Market." *Rev. Econ. Studies* 71 (April): 459–81.

Moskowitz, Tobias J., and Annette Vissing-Jorgensen. 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?" *A.E.R.* 92 (September): 745–78.

Ruhnka, John, Howard Feldman, and Thomas Dean. 1992. "The 'Living Dead' Phenomenon in Venture Capital Investments." *J. Bus. Venturing* 7:137–55.

Sørensen, Morten. 2007. "How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital." *J. Finance* 62 (December): 2725–62.

Ueda, Masako. 2004. "Banks versus Venture Capital: Project Evaluation, Screening, and Expropriation." *J. Finance* 59 (April): 601–21.

Vereshchagina, Halina, and Hugo Hopenhayn. 2009. "Risk Taking by Entrepreneurs." *A.E.R.* 99 (December): 1808–30.