American Economic Journal: Microeconomics 2012, 4(4): 172–201
http://dx.doi.org/10.1257/mic.4.4.172

The Evolutionary Basis of Time Preference: Intergenerational Transfers and Sex[†]

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We consider the evolutionary basis of time discounting with intergenerational transfers. We show that the notion of "reproductive value" from biology provides the utility criterion for a parent to optimize the allocation of resources between transfers to offspring and to promote her own survival. This optimization has a natural dynamic programming formulation. We show that younger individuals may well be "too impatient," but older individuals "too patient" in accordance with observations. We compare the allocation of resources under sexual reproduction to that where there is asexual reproduction. Sex distorts time discounting; under plausible conditions, sex increases patience. (*JEL* A12, D91)

Most models in economics take preferences as given and then derive the choices induced by these preferences. In the tradition of a small, but by now, well-established literature, we turn this around, first characterizing the choice behavior that would result from biological evolution, and then supporting this choice behavior with preferences. That is, we identify the preferences that induce evolutionarily optimal choices.

We make the stark assumption that each choice behavior derives from a particular genetic type. Hence, the choices an individual makes during her lifetime are a characteristic that is inherited from her parents. In the simplest case, without sexual reproduction, this inheritance is without modification, so populations can be defined as a group of individuals having the same genes. Populations with different genetic types may grow at different long run rates. Only those types inducing the highest asymptotic population growth rate survive evolution.

We consider here the biological basis of intertemporal utility and time preferences, in particular. Why do we discount the future at all? What accounts for how much we discount the future and for the age profile of discount rates?

Perhaps the most basic biological model suggests that we should discount the future at the sum of the rate of population growth and the rate of mortality. See Robson and Samuelson (2007), for an example. To set the stage for the present paper, we present an example to exhibit this basic result. To simplify matters, we

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[†] To comment on this article in the online discussion forum, or to view additional materials, visit the article page at http://dx.doi.org/10.1257/mic.4.4.172.

consider here a direct trade-off between current and future offspring. In the main model of the paper with transfers, we consider a trade-off between the survival probabilities of newborns and the survival probability of the parent to the next period. However, the fundamental considerations are the same.

Example 1: Suppose that an individual survives for sure from age zero to age one, and survives with probability p from age one to age two, but then dies. She has a resource endowment of I which she can split between ages one and two. In each of these periods, if she is alive, she transforms her endowment into offspring according to a non-negative, continuously differentiable, strictly increasing and strictly concave reproduction function u. Assume that $u'(x) \to \infty$ as $x \to 0$. A gene is identified with a choice rule $x \in [0, I]$, the use of the endowment in the first period, so the endowment available in the second period is $I - x$. Offspring use the same rule as their parent.

Fix a choice, x, and denote the number of individuals who are one year old at time t by y_t. The following equation recursively defines the law of motion of y_t:

$$y_{t+1} = u(x)\, y_t + pu(I - x)\, y_{t-1}.$$

Indeed, the number of one-year-old individuals at time $t + 1$ who had one-year-old parents at birth is $u(x)\, y_t$. Similarly, $pu\,(I - x)\, y_{t-1}$ is the number of one-year-old individuals at time $t + 1$ whose parents were two years old when they were born.

Dividing both sides by y_t yields

$$\frac{y_{t+1}}{y_t} = u(x) + pu(I - x)\, \frac{y_{t-1}}{y_t}.$$

It is not hard to show that there exists a value of $\lambda > 0$ such that $y_{t+1}/y_t \to \lambda$ as $t \to \infty$, no matter what the initial proportions of one-year-old and two-year-old individuals might be in the population. In the limit, therefore, the previous equation can be rewritten as

$$\lambda^2 = \lambda u(x) + pu(I - x).$$

Let $\lambda(x)$ denote the unique positive solution of this equation for λ, that is,

$$\lambda(x)^2 = \lambda(x) u(x) + pu(I - x).$$

Let x^* denote the choice generating the largest possible growth rate and set $\lambda^* = \lambda(x^*)$. There exists a solution for x, which is unique and interior.[1] Hence, x^* satisfies the

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[1] This is not hard to show, but formal proofs are omitted for brevity. Rigorous proofs of all the necessary formal properties are provided for the general models with transfers, with or without sex.

first-order condition $\lambda'(x^*) = 0$. Differentiating the previous displayed equation and rearranging, we obtain

$$u'(x^*) = pu'(I - x^*)/\lambda^*.$$

This equation implies that the optimal decision x^* must be the unique solution of[2]

$$\max_x u(x) + \frac{pu(I-x)}{\lambda^*}. \tag{1}$$

Expression (1) means that the resource allocation that maximizes the population growth factor, which is the basic biological problem, must also be the allocation that maximizes the expected discounted number of offspring, with the discount factor equal to the probability of survival divided by the *maximal* population growth factor.[3] This latter problem is closely related to the simplest conventional criterion for intertemporal choice, expected discounted utility, where utility is identified with fertility, and where future fertility is deflated by the growth rate divided by the probability of survival. Equivalently, we have found that the *pure* rate of time preference is equal to the rate of mortality plus the rate of population growth.[4] The intuition behind the contribution of population growth is the following. If the population is in steady state growth, with growth factor λ, the value of an expected offspring tomorrow is less, by this factor of λ, than the value of an expected offspring today.[5] This is because one offspring today can herself have λ offspring tomorrow, so that λ offspring produced tomorrow is equivalent to just 1 today.

Many nonhuman species seem to be shaped by a discount rate, one of whose components is mortality. For example, birds typically suffer lower rates of predation than do comparably-sized ground-dwelling mammalian species. As an apparent consequence, they invest more in somatic maintenance and may live even longer than implied directly by the lower predation. Perhaps also as a consequence, birds invest heavily in the rearing of offspring, with the involvement not merely of the female, but also of the male, whereas male involvement is rare in ground-dwelling mammals.[6]

It is hardly surprising that mortality would also influence time discounting in humans, and well-known (see Irving Fisher 1930, for example). The effect of

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[2] Given this uniqueness, the solution here will be supported by other criteria. We address this issue in Example 2 below.

[3] What is the implied attitude to risk in this example? Individuals are risk-neutral in *offspring*, since the fertility function here is best interpreted as *expected* offspring. However, it follows readily that Expression (1) is strictly concave in the total endowment, I, so individuals are strictly risk-averse in resources. The Bernoulli utility function here is a biological production function. These observations generalize straightforwardly to the models in the present paper, given all risk is idiosyncratic.

[4] If, that is, $p = e^{-\delta}$, where δ is the implied continuous-time mortality rate, and $\lambda = e^g$, where g is the implied continuous-time population growth rate, then the pure rate of time preference is $\ln\lambda - \ln p = g + \delta$.

[5] For the purpose of this explanation, mortality is included in the calculation of expected offspring.

[6] Even closer to home is that arboreal mammals live longer than do comparable ground dwelling mammals. Arboreality reduces predation, and has been proposed as the original circumstance leading to greater longevity in primates. See Shattuck and Williams (2010).

population growth on discounting is a recent insight provided by a biological approach. This component of discounting should be constant across all ages, thus providing some support for the simplest of all economic criteria for evaluating intertemporal consumption, namely the sum of discounted utility, where the discount factor is constant.

However, this basic result is not readily squared with observations on rates of time discounting. That is, the average rate of human population growth during the 1.8 million years of our evolutionary history must have been, as a matter of arithmetic necessity, only imperceptibly greater than 0. Plausible estimates of hunter-gatherer mortality rates range from 1 percent for 10 year olds to 4 percent for 60 year olds; but these seem rather lower than plausible estimates of the pure rate of time discount. See Gurven and Kaplan (2007) for these mortality estimates; see Anderson et al. (2008) for experimental estimates of the pure rate of time preference.

One of the contributions of the current paper is to evaluate a candidate for closing this gap that was proposed in a seminal paper by Rogers (1994). This candidate is sexual reproduction in the context of intergenerational transfers. Rogers (1994) argued as follows. Consider a 25-year-old woman who can invest resources to benefit her current newborn. For simplicity, suppose the return from the investment is received 25 years from now, when her newborn will also be 25. Rogers (1994) further supposes that this investment problem has an interior solution. The marginal value of resources will be the same to her offspring 25 years from now as it is to the mother now. However, from the current mother's point of view, sex deflates the importance of her offspring by a factor of $\frac{1}{2}$. In the simplest case of "haploid" sex, this is because this offspring will be a carbon copy of her mother with probability $\frac{1}{2}$, but will be a carbon copy of her father otherwise. Given zero population growth, and apparently abstracting from mortality, a unit of resources 25 years from now will be worth $\frac{1}{2}$ as much to the mother as it is worth today, and Rogers (1994) uses that as the basis for calculating a plausible rate of time preference of around 2 percent.

There are a number of difficulties with the Rogers (1994) analysis that are discussed by Robson and Szentes (2008). One that is easy to outline is that it cannot be true that all these "same age transfers" involve interior solutions. After all, the same argument as above but now applied to a 30-year-old mother contemplating an investment to favor her newborn 30 years from now would imply a lower rate of time preference. In order to address directly the difficulties with Rogers (1994) model, Robson and Szentes (2008) developed an example that permitted same age transfers. However, this requirement made the example awkward enough that it could not shed light on the core claim of Rogers (1994)—that sex is a key factor leading to impatience. The present paper develops a model that is more tractable and therefore illuminating by not being required to allow same age transfers. Although the model is then no longer directly comparable to Rogers (1994), it is an inherently plausible view of transfers and sex. In our model, however, sex may well reduce impatience. Thus sexual reproduction is not guaranteed to close the gap between typical rates of time preference and typical mortality rates.

On the other hand, the current paper provides another way of closing this puzzling gap. Indeed, we address a more refined view of the puzzle. In this more refined view, it is largely children and younger adults who are too impatient relative to mortality rates and the population growth rate. We argue that younger adults are too impatient in this sense because rising resources with age are likely to decrease the marginal productivity of resources transferred to offspring. For children, the model disconnects their time discounting from the mortality rate or the growth rate, so they also may be too impatient.

This more refined view of the puzzle also involves a reversal of the typical gap. That is, older adults are, at the same time, too patient, with moderate rates of time discounting that are too low relative to their increasingly high mortality rates and the population growth rate. The model gives an explanation of this that is the flip side of the explanation for younger adults. That is, since older adults experience decreasing output, this would likely reduce transfers to offspring, and so increase the marginal productivity of such transferred resources.

The above scenario involves time discounting that is is denominated in resource terms, rather than in terms of utility. That is, our measure of time discounting corresponds to the marginal rate of intertemporal substitution in consumption in a conventional model rather than be the usual "pure" rate of time preference, denominated in utility terms. Our model would apply directly to a foraging society in which there were limited opportunities for intertemporal substitution. Time discounting denominated in resource terms would have been shaped by the age profile of energetic income. However, this analysis might continue to apply in a modern setting in which capital markets exist, as long as they remain rather imperfect. Consumption would then continue to track income, as indeed is observed. See Browning and Lusardi (1996), for example. In such a modern setting, it might well then remain the case that younger adults are "too impatient" and older adults "too patient," in the above sense.

Finally, our analysis provides formal insights. We first derive the appropriate notion of "reproductive value" for our model of transfers. This represents a generalization of the simple notion of this, originally due to Ronald A. Fisher (1930), which latter is the expected discounted value of offspring from each age forwards, conditional on survival to that age, where the population growth factor is used to discount the future. This simple notion of reproductive value is only appropriate in contexts in which offspring are homogeneous.[7] The most general notion of reproductive value stems from the relative contribution of each type of offspring to the population in the distant future. In our case, even with heterogeneous offspring, the generalized reproductive value remains an appropriate eigenvector. This approach lends itself to further generalization. A key formal result of the paper is to illuminate a close connection between utility functions and this appropriate notion of reproductive value. The utility function at each adult age is again the expected discounted sum of the *value* of descendants. Although this results in a substantially different expression from that of Ronald A. Fisher (1930), the population growth factor still discounts the future.[8] Optimal choice at each age still involves maximizing current reproductive value. Since current reproductive value derives from the reproductive value of

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[7] A technical flaw in the Rogers model was the application of this simple notion of reproductive value in a context with heterogeneous offspring.

[8] Reproductive value could indeed still expressed in terms of the expected discounted sum of the *value of newborns*. Since newborns are heterogeneous, however, this is not equivalent to the expected discounted sum of the *number of newborns*, as in Ronald A. Fisher (1930).

offspring and the reproductive value of the parent, both one period ahead, lifetime optimal choice solves a simple dynamic programming problem.

A. *A Map of the Paper*

Section I presents a benchmark model that is a stepping stone for our model of transfers. There is an asexual species in which individuals live an arbitrary number of periods. In this benchmark model, we obtain the appropriate reproductive values, which arise as an eigenvector, in a fashion reminiscent of shadow prices in the von Neumann growth model. These values are the expected discounted value of future fertility, conditional on each age, where the discount factor is the population growth factor. How these reproductive values work as utility functions is illustrated by considering a basic biological trade-off between the number of offspring and survival to the next period. The simplification that all offspring are identical is the crucial simplifying feature of this benchmark model. We conclude Section I with an example that considers the circumstances under which the utility criterion is essentially uniquely determined. This involves introducing suitable idiosyncratic random shocks.

In Section IIA, we develop a model of transfers in which offspring cannot be identical, but vary in quality. We then examine the optimal choice of such quality by the parent or parents of each age. In Section IIA, there is an asexual species in which parents trade-off their own survival to the next period against the survival of their newborns. The differential survival of offspring across parental ages means that newborns are heterogeneous. In the presence of such intergenerational transfers, with differential offspring quality, it is not appropriate to maximize the discounted sum of expected offspring. However, reproductive value again arises in the same general way as in the benchmark model as an eigenvector. In addition, we retain the intuitively pleasing result that optimal choice by an individual of each age maximizes this reproductive value.

In Section IIA, we also derive the implications of intergenerational transfers for discounting. We show that the marginal rate of intertemporal substitution for prereproductive individuals is dissociated from the sum of the mortality rate and the population growth rate. For reproductive adults, the marginal rate of intertemporal substitution reflects, as one of its components, this sum. The other component reflects the motive to make transfers to newborns, and this is likely to increase time discounting for younger adults, but decrease it for older adults, in rough agreement with observations.

We introduce sexual reproduction into our model in Section IIB. Sexual reproduction implies that the choices made by the two parents interact, so that the situation is now a game. This has substantial implications for predicted behavior. The reproductive values derive from the pattern of fertility and survival in the same way as for the asexual species. An individual of each age still maximizes a utility function that derives from reproductive value, with the key difference from the asexual case that sexual individuals deflate the importance of the survival of their offspring by a factor of $\frac{1}{2}$. This is because if the gene has frequency zero in the population, then the probability of the other parent having the same gene is zero, so the probability an offspring has the same gene is $\frac{1}{2}$. This is true even though, in the evolutionary equilibrium, all individuals have the same gene.

The following results are established. Every adult systematically skews her allocation of resources to favor her own survival at the expense of her offspring's survival. This is true in a myopic sense, that is, holding constant the reproductive value of the adult in the next period. This distortion will inevitably affect these reproductive values. However, even when the implications of sex for reproductive values are appropriately allowed for, the sexual type increases all adult survival rates at the expense of all newborn survival rates. Sex then unambiguously reduces the quality of offspring in this sense. However, this distortion does not entail greater impatience. Rather, sex may well imply too much patience, since greater adult survival militates in this direction.

Section III concludes by discussing issues of interpretation, extensions that would be of definite interest, despite varying degrees of feasibility, and an application to the global warming debate.

I. A Benchmark Model without Transfers

Consider an asexual species in which each individual lives to a maximum age of T periods. At age $\tau = 0, \ldots, m-1$, she has $u_\tau = 0$ offspring; at age $\tau = m, \ldots, T$, she has $u_\tau > 0$ expected offspring. In this benchmark model, but not subsequently, the offspring produced by parents of differing ages are identical. Finally, each individual survives from age τ to age $\tau + 1$ with probability $p_\tau \in (0, 1)$, for $\tau = 0, \ldots, T-1$.

The parameters just described in general will eventually be taken to depend on the choice made by the individuals, so the issue is to derive the optimal choice. We consider a simple explicit choice problem below. As a necessary building block, we first consider the implications of a particular fixed set of parameters for the growth rate of population.

The population then evolves as

$$\mathbf{n}^{t+1} = \mathbf{n}^t \mathbf{L},$$

where $\mathbf{n}^t$ is a row-vector describing the population at date t so that $\mathbf{n}^t = (n_1^t, \ldots n_T^t)$ where n_τ^t is the number of individuals of age $\tau = 1, \ldots, T$ at date t.[9] Also $\mathbf{L}$ is the Leslie matrix

$$\mathbf{L} = \begin{bmatrix} p_0 u_1 & p_1 & 0 & . & . & 0 \\ p_0 u_2 & 0 & p_2 & 0 & .. & 0 \\ p_0 u_3 & 0 & 0 & p_3 & .. & . \\ \ldots & .. & . & . & . & 0 \\ p_0 u_{T-1} & 0 & . & . & 0 & p_{T-1} \\ p_0 u_T & 0 & . & . & . & 0 \end{bmatrix}.$$

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[9] It is convenient to consider newborns, with $\tau = 0$, only implicitly here. This asymmetric reduction in the number of age classes generates the need for a little algebra here and there, but saves a lot elsewhere, and is especially helpful in the following sections that treat transfers.

All of these columns except the first simply describe how the number of individuals of age $\tau = 2, \ldots, T$ at date t is the number of individuals of age $\tau - 1$ at date $t - 1$, allowing for survival, $p_{\tau-1}$. The first column describes how the number of age 1 individuals is the total number of newborns one period ago, $\sum_{\tau=1}^{T} u_\tau n_\tau^{t-1}$, allowing for survival, p_0.

The Perron-Frobenius Theorem implies that the system settles into steady state growth with the growth factor $\lambda > 0$ being the unique dominant eigenvalue.[10] Furthermore, the left eigenvector $\mathbf{q} >> \mathbf{0}$ (taken as a row vector) gives the limiting population proportions and the right eigenvector $\mathbf{v} >> \mathbf{0}$ (so $\mathbf{v}^T$ denotes the associated column vector) gives the relative values of each age class. That is, we have

$$\mathbf{qL} = \lambda\mathbf{q}, \quad \mathbf{Lv}^T = \lambda\mathbf{v}^T \quad \text{and} \quad |\mathbf{L} - \lambda\mathbf{I}| = 0.$$

Note that $\mathbf{q}$ and $\mathbf{v}$ are only determined up to a multiplicative constant. A straightforward proof by induction shows that the characteristic equation is

$$(2) \qquad 1 = \frac{p_0 u_1}{\lambda} + \cdots + \frac{p_0 \cdots p_{T-1} u_T}{\lambda^T},$$

which indeed is the Euler-Lotka equation and can be obtained more directly by straightforward arguments from the steady state.

The left eigenvector gives the relative proportions of individuals in each age class in the steady state, as is familiar in demography. It can be taken to be, for example, $\mathbf{q} = \left(\frac{p_0}{\lambda}, \frac{p_0 p_1}{\lambda^2}, \ldots, \frac{p_0 \cdot\cdot p_{T-1}}{\lambda^T}\right)$.[11]

The right eigenvector satisfies

$$(3) \qquad v_\tau = u_\tau + \frac{p_\tau v_{\tau+1}}{\lambda}, \quad \tau = 1, \ldots, T-1,$$

the solution of which can be taken as

$$(4) \qquad v_\tau = u_\tau + \frac{p_\tau u_{\tau+1}}{\lambda} + \cdots + \frac{p_\tau \cdot \cdot p_{T-1} u_T}{\lambda^{T-\tau}}, \quad \tau = 1, \ldots, T,$$

which are the reproductive values as defined by Ronald A. Fisher (1930). That is, v_τ is the expected discounted sum of future fertility, conditional on being alive at age τ, where the expectation includes the probability of survival to each future age, and where the discount factor is the population growth factor.

How would evolution choose between arbitrary profiles of survival probabilities and fertilities, with each profile of the form $\{p_\tau, u_{\tau+1}\}_{\tau=0}^{T-1}$? The theory of evolutionary choice can be developed in the same way as it is conventional to describe

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[10] See Seneta (1981). It is enough that there exist two ages τ and $\tau' = \tau + 1$ such that u_τ and $u_{\tau'}$ are strictly positive.

[11] This form of $\mathbf{q}$ would arise from normalizing $q_0 = 1$ if the newborns were explicitly included.

preferences in consumer theory. That is, evolutionary preferences are independent of the set of options considered. The most general biological representation of intertemporal preferences is the growth factor λ. Often, however, a more useful representation from an economic perspective is the right-hand side of the Euler-Lotka equation, equation (2). These are connected in that it is necessary that a growth maximizing profile of demographic characteristics p_τ and $u_{\tau+1}$ for $\tau = 0, \ldots, T-1$ also maximize the right-hand side of this equation, given the *optimal* growth factor λ.[12]

To illustrate this approach in a concrete fashion, and one congenial to economists, since it involves allocating a scarce resource, consider a very basic biological trade-off—that between the number of offspring and own survival. Considering this trade-off allows us to remain within the benchmark model, since all offspring remain identical. (The trade-off considered in the model of transfers, on the other hand, forces us to consider heterogeneous offspring.) Suppose now that each adult individual of some particular age $\tau \in \{m, \ldots, T\}$ has a resource budget or income $I_\tau > 0$. This budget is to be divided between resources used to produce offspring, $r_\tau \geq 0$, via the function $u_\tau(r_\tau)$, and resources used to promote survival to the next period, $s_\tau \geq 0$, via the function $p_\tau(s_\tau)$, so that $I_\tau = r_\tau + s_\tau$. Suppose the functions $u_\tau(\cdot)$, and $p_\tau(\cdot)$ are non-negative, continuously differentiable, with a strictly positive derivative everywhere, and strictly concave everywhere.

The reproductive values permit a nice view of optimal choice, as follows. From the Euler-Lotka equation, equation (2), and the equation determining v_τ, equation (4), the first-order condition for the optimal allocation of resources by an individual of age τ must solve the following problem

$$\max_{\substack{r_\tau, s_\tau \geq 0\\ r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau)$$

$$\equiv \max_{\substack{r_\tau, s_\tau \geq 0\\ r_\tau + s_\tau = I_\tau}} \left(u_\tau(r_\tau) + \frac{p_\tau(s_\tau)}{\lambda}\left[u_{\tau+1} + \frac{p_{\tau+1}u_{\tau+2}}{\lambda} + \cdots + \frac{p_{\tau+1}\cdots p_{T-1}u_T}{\lambda^{T-\tau-1}}\right]\right).$$

This explicitly spells out the utility criterion relevant to this age τ individual. This can be more compactly expressed as

$$\max_{\substack{r_\tau, s_\tau \geq 0\\ r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau) \equiv \max_{\substack{r_\tau, s_\tau \geq 0\\ r_\tau + s_\tau = I_\tau}} u_\tau(r_\tau) + \frac{p_\tau(s_\tau)}{\lambda} v_{\tau+1}.$$

In either case, this age τ individual solves the problem of maximizing her reproductive value $v_\tau(r_\tau, s_\tau)$ which is then interpreted as her utility function.

This model could be generalized to consider optimal choices like this by individuals of all ages, making the choice then of whether fertility is zero or strictly positive fully endogenous. Such a model could then endogenize the transition from childhood, with zero fertility, to adulthood, with positive fertility. This would not be a derivation from first principles, however, since it would be predicated on

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[12] For suppose equation (2) holds, thus determining the value of λ. but that there exists a demographic profile that raises the right-hand side of equation (2). It follows that λ can then be raised above its original value.

age-varying income and the age-varying survival functions. See Kaplan and Robson (2009), for a model that generates all of our key demographic characteristics without relying on time-varying parameters or functions.

One natural question concerning this model and the subsequent ones is: Given that the earth is of a limited size, surely it is not possible that the maximal λ is greater than 1? That is, surely finite land, for example, forces the maximal λ to be no greater than 1? One way that the model can be adapted to this observation is to suppose that survival rates p_τ are decreasing functions of total population N, say. This dependence might plausibly be such that growth factors greater than 1 are possible at low values of N, but that growth is inevitably choked off as N grows. In the limit then a growth factor of exactly 1 will emerge in an endogenous fashion. We will return to this issue when comparing the models of transfers with and without sex.

Another natural question that arises with respect to the above model and more generally is: Since there is only one optimal choice, surely the given criterion is not the only one that supports it? We use the following example to sketch the intuition that the addition of suitable noise means that the criterion is essentially uniquely determined. This example is a simplification of the benchmark model described above, except for the source of idiosyncratic noise.

Example 2: Suppose then that $T = 2$ and that fertility at age 1 is a non-negative, continuously differentiable, strictly increasing, and strictly concave function of the resources allocated for that purpose, r, say, so that fertility is given by $u_1(r)$. If the resources used to promote survival from age 1 to age 2 is s, then this survival probability is $p(s)$, where $p(\cdot)$ is assumed to be continuously differentiable, strictly increasing and strictly concave. Suppose survival from age 0 to age 1 is certain, and that fertility at age 2 is u_2.[13]

Suppose now that the resource endowment is a random variable, given by $\tilde{I}$, which has support $[0, \infty)$. Furthermore, the effectiveness of resources in promoting the survival of offspring is also a random variable given by $\tilde{\alpha}$ also with full support $[0, \infty)$. These random variables are independent of each other, and also independent across individuals.[14] The budget constraint then has the form $r + \tilde{\alpha} s = \tilde{I}$.

The individual is taken to be aware of the joint realization (α, I) of the random variable pair $(\tilde{\alpha}, \tilde{I})$ and to then choose the resource allocation $(r(\alpha, I), s(\alpha, I))$, where $r(\alpha, I) + \alpha s(\alpha, I) = I$.

The basic question then is: To what extent is the appearance of the functions $u_1(\cdot)$ and $p(\cdot)$ in utility uniquely determined by the need to account for the evolutionarily optimal choices to be made here by the individual?

It is first necessary to address the question: In the presence of this random variable, and given a large population, what is the growth rate of a type that follows an

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[13] This specification is consistent with the example of choice considered above. An alternative specification that would agree more closely with Example 1 would suppose that the survival probability, p, is fixed but that fertility at age 2 is endogenous, given by $u_2(s)$, say. This makes only notational differences to the argument here.

[14] The two random variables do not need to be independent of each other; it is enough that they have full support, namely $[0, \infty)^2$. However, see Robson and Samuelson (2009) for an investigation of the substantial consequences of relaxing the requirement of independence across individuals.

arbitrary allocation rule $(r(.,.), s(.,.))$? Expected offspring of an age 1 individual is $E_{\tilde{\alpha},\tilde{I}}[u_1(r(\tilde{\alpha}, \tilde{I}))]$; that of all age 2 individuals is $E_{\tilde{\alpha},\tilde{I}}[p(s(\tilde{\alpha}, \tilde{I}))]u_2$, where this expectation is formed from a point of view at age 0, and so includes the survival probability from age 1 to age 2 in particular. Since all the risk here is idiosyncratic and the population is assumed to be large, the long run growth factor λ satisfies equation (2) so that

$$1 = E_{\tilde{\alpha},\tilde{I}}\left[u_1(r(\tilde{\alpha}, \tilde{I})) + \frac{p(s(\tilde{\alpha}, \tilde{I}))u_2}{\lambda}\right].$$

The best allocation rule maximizes λ in this version of the Euler-Lotka equation. It is then clear that the optimal allocation rule solves

$$\max_{\substack{r,\, s \geq 0 \\ r + \alpha s = I}} \; u_1(r) + \frac{p(s)u_2}{\lambda}, \tag{5}$$

for each pair of realizations (α, I) of the random variables $(\tilde{\alpha}, \tilde{I})$.

This is now entirely analogous to a familiar problem from consumer theory. Problem (5) gives rise to fully specified "demand functions" $r(\alpha, I)$ and $s(\alpha, I)$ that represent the behavior that must be generated for evolutionary optimality. Using this demand analogy, it follows that these functions can be "integrated" to obtain a utility function that is unique up to an arbitrary monotonic transformation. That is, the only overall criteria that generates the same required behavior must be of the form $\psi\left[u_1(r) + \frac{p(s)u_2}{\lambda}\right]$, for some strictly increasing function ψ. If ψ is twice continuously differentiable, and the overall criterion is required to remain additively separable, the only flexibility left is to multiply both $u_1(r)$ and $p(s)\,u_2$ by a common positive constant, and to add possibly different arbitrary constants to these functions. In this straightforward sense, the functions $u_1(r)$ and $p(s)\,u_2$ are essentially uniquely identified.

Note how the population growth factor λ that should be applied in equation (5) is derived from average population-wide fertilities and is not customized to the particular individual. That is, an individual who obtains high realizations of $\tilde{\alpha}$ and $\tilde{I}$, for example, should use the same factor λ in her calculations as should an individual with low realizations. Idiosyncratic risk then does not cause varying impatience.

There is no reason to doubt that an analogous argument can also be applied in the model of the next section. That is, not only would the argument generalize to allow for the trade-off there between survival of newborns and survival of the adult to the next period, but to allow an arbitrary number of ages as well.

II. The Model of Transfers

The point of the Rogers (1994) approach was to examine how sexual reproduction was a source of impatience when transfers can be made from parents to offspring. Particular problems with Rogers' formulation were examined by Robson and Szentes (2008). In this section, we reexamine this issue in a more natural model that is not constrained by the need to allow for "same-age" transfers as in Rogers (1994). Indeed, we consider here the simplest kind of transfer—one from a parent to a newborn that increases the probability of that newborn surviving to the next period. The opportunity cost to the parent of this transfer is a reduction in the probability of the parent surviving to the next period. Resources cannot be explicitly saved or carried forward at all, also in contrast to Rogers (1994). This is defensible on the grounds of realism, since there were no obvious direct ways of making commodity trade-offs across widely separated dates in hunter-gatherer societies prior to agriculture. At the same time, it is crucial that we ultimately consider how such individuals might once have made such trade-offs implicitly, and *would* make explicit commodity trade-offs once they became available. We show that these implicit trade-offs may well mean that children and younger adults have rates of time preference that exceed the sum of the mortality rate and the population growth rate; but older adults have rates that are less than this sum.

A. *Transfers without Sex*

Consider again an asexual species. Suppose now that individuals of ages $\tau = 1, \ldots, T$ have incomes $I_\tau > 0$. Newborns of age 0 have income 0. Each adult individual of age $\tau = m, \ldots, T$ transfers an amount $r_\tau \geq 0$ to each of her $u_\tau > 0$ newborn offspring, keeping $s_\tau \geq 0$ to promote her own survival to age $\tau + 1$. It is now generally inescapable that offspring from parents of different ages will be different. An offspring who is the beneficiary of a larger transfer and so survives with higher probability has higher "quality" in that sense than one with a lower transfer. This heterogeneity must be taken into account in the appropriate notion of reproductive value. The budget constraint is $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, \ldots, T$. Children of age $\tau = 1, \ldots, m - 1$ have no offspring so that $u_\tau = 0$ and they set $r_\tau = 0$ and $s_\tau = I_\tau$. In addition, $s_T = 0$ so that $r_T = I_T / u_T$.[15]

The effect of the transfers is to promote the survival of newborns for one period. That is, this survival probability is endogenous, and given by $p_0(r_\tau)$, for each offspring of an age $\tau = 1, \ldots, T$ parent. Survival of each age τ parent to the next period is given by $p_\tau(s_\tau)$. The functions $p_\tau(\cdot)$, $\tau = 0, \ldots, T$ are non-negative, continuously differentiable, strictly concave, with a strictly positive derivative everywhere, and where this derivative tends to infinity at 0. In this formulation, we assume for simplicity that the fertilities u_τ for $\tau = 1, \ldots, T$ are fixed, in contrast to the choice considered in the context of the benchmark model.

Again we have

$$\mathbf{n}^{t+1} = \mathbf{n}^t \mathbf{L},$$

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[15] For simplicity, this model does not consider transfers from parents who are no longer fertile to their older children or grandchildren. This interesting issue is taken up in Section III.

where $\mathbf{n}^t$ is row-vector describing the adult population at date t so that $\mathbf{n}^t = (n_1^t, \ldots, n_T^t)$, and where the Leslie matrix is now[16]

$$\mathbf{L} = \begin{bmatrix} p_0(r_1)u_1 & p_1(s_1) & 0 & . & .. & 0 \\ p_0(r_2)u_2 & 0 & p_2(s_2) & 0 & .. & 0 \\ p_0(r_3)u_3 & 0 & 0 & p_3(s_3) & .. & . \\ \ldots & . & . & . & .. & 0 \\ p_0(r_{T-1})u_{T-1} & 0 & . & . & 0 & p_{T-1}(s_{T-1}) \\ p_0(r_T)u_T & 0 & . & . & . & 0 \end{bmatrix}.$$

All the columns except the first have a similar simple interpretation to that for the benchmark model. The first column describes how the number of individuals of age 1 at date t arises from the newborns of all individuals of age $\tau = 1, \ldots, T$ at date $t-1$ as the sum $\sum_{\tau=1}^{T} p_0(r_\tau)u_\tau n_\tau^{t-1}$.

The Euler-Lotka equation, or, equivalently, the characteristic equation for $\mathbf{L}$, namely $|\mathbf{L} - \lambda\mathbf{I}| = 0$, is now

$$(6) \qquad 1 = \frac{p_0(r_1)u_1}{\lambda} + \frac{p_0(r_2)p_1(s_1)u_2}{\lambda^2} \cdots + \frac{p_0(r_T)p_1(s_1)\ldots p_{T-1}(s_{T-1})u_T}{\lambda^T}.$$

The most basic view of preferences here, as before, is λ, but a useful representation is often the right-hand side of this equation. That is, maximizing this expression, taking the *optimal* value of the growth factor, λ, as parametric, is a necessary condition for maximizing this growth factor.[17]

The limiting population proportions, $\mathbf{q}$, can be taken to be $\mathbf{q} = \left(1, \frac{p_1(s_1)}{\lambda}, \ldots, \frac{p_1(s_1)\ldots p_{T-1}}{\lambda^{T-1}}\right)$, with the normalization that $q_1 = 1$. The reproductive values again satisfy $\mathbf{L}\mathbf{v}^T = \lambda\mathbf{v}^T$. That is, with the normalization that $v_1 = 1$,[18]

$$(7) \quad v_\tau = \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \quad \text{for } \tau = 1, \ldots, T-1, \quad \text{with } v_T = \frac{p_0(I_T/u_T)u_T}{\lambda}.$$

These equations straightforwardly relate the value of an individual of age τ to the value of her offspring, derived as the expected value of these one period ahead, plus

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[16] The advantage of suppressing explicit treatment of newborns is now significant. If newborns were included explicitly, that is, there would be T different types of them, one for each possible parental age.

[17] For suppose that the allocation $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ does *not* maximize the right-hand side of equation (6), where equation (6) is itself satisfied, thus determining λ. There must then exist an alternative allocation that raises the right-hand side of equation (6) above 1. This implies that λ can be increased to restore equality, and that the original value could not have been optimal.

[18] For expositional economy, children, who have $u_\tau = 0$, for $\tau = 1, \ldots, m-1$, are included in this formulation.

the expected value of the individual herself one period ahead. It is easy to solve these equations by backwards recursion to obtain each v_τ as follows

$$(8)\quad v_\tau = \frac{1}{\lambda}\left\{p_0(r_\tau)u_\tau + \frac{p_0(r_{\tau+1})p_\tau(s_\tau)u_{\tau+1}}{\lambda} + \cdots + \frac{p_0(r_T)p_\tau(s_\tau)\ldots p_{T-1}(s_{T-1})u_T}{\lambda^{T-\tau}}\right\},$$

for $\tau = 1, \ldots, T$.[19]

The expression in equation (8) in particular details the reproductive value of an age $\tau = m, \ldots, T-1$ adult individual. These individuals face nontrivial choice problem since they have both newborn offspring *and* an endogenous probability of survival to the next period. Each adult individual of age $\tau = m, \ldots, T-1$ maximizes her reproductive value which we then interpret as her utility function.

This utility function has some rather familiar properties—additive separability and the appearance of a constant discount factor, λ, for example. In addition, the term $p_0(r_\tau)$ is analogous to a age-invariant felicity function. The less familiar properties of this expression derive from the interpretation of the $p_\tau(s_\tau)$ for $\tau = 1, \ldots, T$, as survival probabilities. In the first place, these probabilities are taken to be subject to choice here. Further, it is the product of these probabilities between the current age and any future age that enters the above criterion.

The recursive relationship in equation (7) permits a simple dynamic programming view of these values that is tailored to the current model by highlighting the consequences of the current choice.[20]

THEOREM 1: *In the present model of transfers without sex, the unique optimal allocation of resources by adults of age* $\tau = m, \ldots, T-1$ *is the unique solution to the dynamic programming problem*

$$(9)\qquad \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau),$$

where equation (7) *yields the* v_τ, *and where Euler-Lotka equation, equation* (6), *yields* λ.

PROOF:

Dynamic programming implies that the unique choices of age $\tau = m, \ldots, T-1$ adults described in equation (9) and equation (7) maximize the RHS of the Euler-Lotka equation, equation (6), for an arbitrary $\lambda > 0$. If $V(\lambda)$ denotes this maximized value of the RHS of equation (6), then it follows readily that (i) $V(\cdot)$ is continuously differentiable, with $V'(\lambda) < 0$, for all $\lambda > 0$; that (ii) $V(\lambda) \to \infty$, as $\lambda \to 0$; and that (iii) $V(\lambda) \to 0$, as $\lambda \to \infty$. (i) Using the envelope theorem, where $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ is the optimal profile, it

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[19] By suppressing explicit treatment of newborns, we finesse the issue of their heterogeneity. If they were not suppressed, that is, we would need T additional reproductive values.

[20] See Houston and McNamara (1999) for an excellent treatment of such dynamic programming results from a biological point of view. See, in particular, McNamara, Houston, and Webb (1994) for an application to kin selection.

follows that $V'(\lambda) = -p_0(r_1)u_1/\lambda^2 - 2p_0(r_2)p_1(s_1)u_2/\lambda^3 - \cdots < 0$. (ii) Since $V(\cdot)$ is bounded below by the value the right-hand side of equation (6) has for an arbitrary allocation, it also follows that $V(\lambda) \geq p_0(I_m)p_1(I_1)\ldots p_{m-1}(I_{m-1})u_m/\lambda^m \to \infty$, $\lambda \to 0$. (iii) Let $\bar{u} = \max_\tau\{u_\tau\}$. Since $p_0(\cdot), p_\tau(\cdot) \leq 1$, it follows that $V(\lambda) \leq \bar{u}/\lambda\{1 + 1/\lambda + 1/\lambda^2 + \cdots\} = \bar{u}/(\lambda - 1)$, assuming $\lambda > 1$. That is, $V(\lambda) \to 0$ as $\lambda \to \infty$.

Hence there exists a unique $\lambda^* > 0$ satisfying $V(\lambda^*) = 1$. This is the maximum feasible growth factor, since if $\lambda > \lambda^*$, so that $V(\lambda) < 1$, then there is no resource allocation profile such that λ satisfies equation (6).[21]

Given the properties of the survival functions $p_\tau(\cdot)$, for $\tau = 0, \ldots, T$, there must be a unique interior solution to the problem of maximizing the right-hand side of equation (6)—that is, with $r_\tau > 0$ and $s_\tau > 0$, for $\tau = m, \ldots, T-1$. The first-order conditions, which are then necessary and sufficient, are

$$p_0'(r_\tau) = p_\tau'(s_\tau)v_{\tau+1}, \tau = m, \ldots, T-1,$$

which, together with equation (6) itself and equation (7), characterize the optimal lifetime allocations of individuals for whom this decision is nontrivial.[22]

The marginal rate of (intertemporal) substitution between adjacent resource income levels I_τ and $I_{\tau+1}$ is given by

$$MRS_\tau = \frac{\dfrac{\partial\lambda}{\partial I_\tau}}{\dfrac{\partial\lambda}{\partial I_{\tau+1}}}, \tau = 1, \ldots, T-1.$$

In the present case, it follows from the Euler-Lotka equation, equation (6), that, for adults of age $\tau = m, \ldots, T-1$, for whom there are transfers to newborns,[23]

$$MRS_\tau = \frac{\lambda p_0'(r_\tau)}{p_\tau(s_\tau)p_0'(r_{\tau+1})}.$$

This expression for *MRS* yields a rate of time preference denominated in resource terms. The *MRS* is the applicable bottom-line exchange rate that would be used by an individual considering the effect of a small intertemporal rearrangement of resources and so is the rate that would be most easily observed in practice.

There is a familiar component of this expression given as $\lambda/p_\tau(s_\tau)$, which corresponds to the "pure rate of time preference." This component is the prediction of the basic model as sketched in Example 1. Lower survival to the next period, as reflected in a higher rate of mortality, is a familiar reason for impatience (since

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[21] That is, under the hypotheses of the theorem, it is not only *necessary* for a profile to be optimal that it maximize the right-hand side of equation (6) and that equation (6) hold, but *sufficient* as well.

[22] That is, these individuals have both newborn offspring and an endogenous probability of survival themselves.

[23] To show this, apply the envelope theorem to equation (6). First note that $\partial\lambda/\partial I_\tau = \partial\lambda/\partial s_\tau = (\partial\lambda/\partial r_\tau)/(1/u_\tau)$. Now we obtain

$$(\partial\lambda/\partial I_\tau)\{p_0(r_1)u_1/\lambda^2 + 2p_0(r_2)p_1(s_1)u_2/\lambda^3 + \cdots\} = p_0'(r_\tau)p_1(s_1)...p_{t-1}(s_{\tau-1})/\lambda^\tau.$$

Since the coefficient of $(\partial\lambda/\partial I_\tau)$ is independent of τ, the desired expression follows.

Irving Fisher 1930, at least). There is also a biological cause of impatience (which has also become more familiar recently)—population growth, as reflected in λ.

Additionally, this expression for the *MRS* accounts for the effect of variation in the marginal product of resources used to promote the survival of newborns. This effect is analogous to how different marginal utilities of consumption affect the *MRS* in a simple standard model of intertemporal choice.

Consider now why younger individuals might be "too impatient " and older individuals "too patient," in terms of the *MRS*. First: Why do younger individuals seem more impatient than implied by the sum of the rate of mortality and the population growth rate? Second: Why, on the other hand, do older individuals seem more patient than this?[24] It is convenient to start with adults.

The component of the *MRS* that goes beyond mortality and population growth is $p_0'(r_\tau)/p_0'(r_{\tau+1})$. If the transfers r_τ increase with age, this term would increase impatience since

$$\frac{p_0'(r_\tau)}{p_0'(r_{\tau+1})} > 1. \tag{10}$$

It is relevant here that average productivity for hunter-gatherers increases until age 45 or so.[25] We then hypothesize that productivity and transfers are correlated. With a reproductive career that starts at 15, say, the model accordingly predicts greater impatience, in terms of the *MRS*, for the age range 15–45, than implied by mortality and population growth.[26]

Once the transfers r_τ decrease with age, on the other hand, this term would then reduce impatience, as in *MRS*, since

$$\frac{p_0'(r_\tau)}{p_0'(r_{\tau+1})} < 1. \tag{11}$$

The flip side of the prediction for younger adults is then that the model predicts that older individuals, whose output is declining, would be "too patient." From the hunter-gatherer data, this prediction would apply to those over 45 years old.[27]

We now complete the picture for younger individuals by considering the impatience of children. For children of age $\tau = 1, \dots, m-2$, we have

$$MRS_\tau = \frac{\lambda p_\tau'(I_\tau) v_{\tau+1}}{p_\tau(I_\tau) p_{\tau+1}'(I_{\tau+1}) v_{\tau+2}}.$$

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[24] See Anderson et al. (2008), and Bishai (2004), for example, for evidence on these two assertions.

[25] See figure 3 in Kaplan and Robson (2009), for example.

[26] The correlation of productivity and transfers concerns the endogenous variable r_τ and it may not hold for all possible specifications. It is important then to verify that there are assumptions on the primitives that ensure this property holds. Suppose, for example, that $u_\tau = u$, a constant, for all $\tau = m, \dots, T$ and that the own survival rates are also independent of age, given by $p_\tau(s) = p(s) = \beta s^\gamma$, where $\beta, \gamma > 0$, and γ is small. In this case, it follows that s_τ must be small (but positive), so that variation in the r_τ must account for most of the variation in the I_τ.

[27] A more general model than that here would allow for transfers to more than one age of offspring, or to grandchildren. This would mean that individuals who were beyond reproductive age would still value the resources they could use to make these transfers. This issue is taken up in the Conclusion.

However, since $u_{\tau} = 0$ for $\tau = 0, \ldots, m - 1$ it follows that

$$v_{\tau} = \frac{p_{\tau}(I_{\tau})v_{\tau+1}}{\lambda}, \tau = 1, \ldots, m - 1,$$

so that

$$MRS_{\tau} = \frac{p'_{\tau}(I_{\tau})p_{\tau+1}(I_{\tau+1})}{p'_{\tau}+1(I_{\tau+1})p_{\tau}(I_{\tau})}, \tau = 1, \ldots, m - 2.$$

This expression for the impatience of children is not directly connected to the survival rate or to the rate of population growth, in contrast to the analogous expression for the impatience of adults.

At the transition from childhood to adulthood, at age $\tau = m - 1$, we have the special case that

$$MRS_{m-1} = \frac{\lambda p'_{m-1}(I_{m-1})v_m}{p_{m-1}(I_{m-1})p'_0(r_m)}.$$

How impatient then are children? It is helpful here to define $\delta_{\tau}(s_{\tau}) = -\ln(p_{\tau}(s_{\tau}))$ as the continuous time mortality rate implied by $p_{\tau}(s_{\tau})$, so $\delta'_{\tau}(s_{\tau}) < 0$. It follows that the impatience of children derives from the change in the derivative of this mortality rate. That is, $MRS_{\tau} > 1$ for $\tau = 1, \ldots, m - 2$ if and only if

$$\delta'_{\tau}(I_{\tau}) < \delta'_{\tau+1}(I_{\tau+1}) < 0. \tag{12}$$

This condition requires that the marginal benefit of resources that decrease the mortality rate is higher for younger ages, and we assume that it is satisfied.

The impatience of children, as in the *MRS*, does not depend directly on mortality rate or population growth, and may then be excessive, complementing the results above for younger adults.[28]

How does the impatience of children depend on age? For $\tau = 2, \ldots, m - 2$, it follows readily that $MRS_{\tau-1} > MRS_{\tau} > 1$ if and only if

$$\sqrt{\delta'_{\tau-1}(I_{\tau-1})\delta'_{\tau+1}(I_{\tau+1})} > -\delta'_{\tau}(I_{\tau}) > 0. \tag{13}$$

We assume this condition holds in order to generate the decreasing pattern of time preference usually ascribed to children. (See Bettinger and Slonim 2007.)[29]

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[28] It plausible but not certain that this is true for infants, since they have extremely high but hard to measure rates of time preference but mortality rates that are high even now, but were still higher in our evolutionary past. The model need not take a stance on this.

[29] An apparently significant omission from the above model concerns the growth of children. That is, children allocate significant resources to somatic growth as well as to survival. Explicit treatment of this is omitted for simplicity and since the focus here is rather on adults. However, including this application for resources may not make a huge qualitative difference. That is, the high marginal product of resources in promoting growth early in life would tend to reduce the resources available for survival. This would reinforce the above observations, since

To sum up the expressions obtained here for the rate of time discounting:

THEOREM 2: *In the current model of transfers without sex, the marginal rate of intertemporal substitution for children, those of age* $\tau = 1, \ldots, m - 2$ *is given by*

$$MRS_\tau = \frac{p_\tau'(I_\tau) p_{\tau+1}(I_{\tau+1})}{p_{\tau+1}'(I_{\tau+1}) p_\tau(I_\tau)}.$$

Under equations (12) *and* (13), *children exhibit a rate of time discounting, from their MRS, that is positive, but decreasing with age. At the transitional age of* $\tau = m - 1$, *a child has*

$$MRS_{m-1} = \frac{\lambda p_{m-1}'(I_{m-1}) v_m}{p_{m-1}(I_{m-1}) p_0'(r_m)}.$$

On the other hand, the marginal rate of intertemporal substitution for adults, those of age $\tau = m, \ldots, T - 1$, *is given by*

$$MRS_\tau = \frac{\lambda p'_0(r_\tau)}{p_\tau(s_\tau) p'_0(r_{\tau+1})}.$$

Adults exhibit impatience, as derived from their MRS, that is at first excessive relative to the mortality and population growth, but then too small, assuming that transfers to newborns track output, as in the discussion of equations (10) *and* (11).

The predictions obtained here apply literally to a hunter-gatherer society. In particular, it is plausible that individuals in such a society would be more impatient than dictated by mortality and population growth when younger, but less impatient when older. How might these predictions translate to a modern setting?

Suppose that what was hard-wired was not the specific fashion that the various functions here depend on their arguments, nor even these arguments themselves necessarily, but just their interpretation as the production and survival of offspring and own survival. These arguments are evolutionarily proximate inputs for the production and nurturing of offspring. The relevance of some of these inputs, such as sex, might have been rather constant over time, only to change rather recently. The relevance of particular commodities, on the other hand, might have changed dramatically. Suppose individuals flexibly and perhaps consciously accounted for how the values of these functions are influenced by their choices.

That is, what this approach gives us is an evolutionary basis for utility, with a time-invariant component, as represented in the above model by the function $p_0(\cdot)$. Now observe that the age profile of individual output in modern societies is qualitatively very similar to that for hunter-gatherers. Moreover, although there is now a market for saving and borrowing, it remains rather imperfect,

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the rate of time preference denominated in resources could still be taken in the form given above, except only that the resource argument would now refer to the resources devoted to survival rather than the total resources available.

perhaps accounting for the observation that consumption still tracks income to a pronounced extent (See Browning and Lusardi 1996). The above analysis might then well apply, perhaps in somewhat less dramatic way and *mutatis mutandis*, to modern economies.

B. *Transfers and Sex*

Reconsider the model of Section IIA after the introduction of sexual reproduction. We derive the new equilibrium allocation of resources for each adult between his/her offspring and his/her own survival. We hypothesize that there may be multiple types, where each type is characterized by a particular age profile of allocations. Mating pairs must be the same age and mating is random with respect to type. We simplify the process of sexual reproduction by supposing that each offspring inherits each of the resource allocation profiles of her parents with probability $1/2$. That is, there is so-called "haploid sex."[30] An equilibrium resource allocation profile will have the property that a small number of mutants with any alternative allocation profile will do worse. We will consider then the growth process for a small number of mutants with an arbitrary resource allocation profile in a population with some given allocation profile.[31]

As before, individuals of ages $\tau = 1, \ldots, T$ have incomes $I_\tau > 0$. Newborns have income zero. Each adult of age $\tau = m, \ldots, T$ now transfers an amount $r_\tau/2$ to each of the $2u_\tau > 0$ newborn joint offspring, keeping s_τ to promote her own survival to age $\tau + 1$. The budget constraint is $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, \ldots, T$. As before, children use all their resources on their own survival, given $u_\tau = 0$, so that $s_\tau = I_\tau$ for $\tau = 0, \ldots, m - 1$. Also $s_T = 0$ so that $r_T = I_T/u_T$. The survival functions $p_\tau(\cdot)$ for $\tau = 0, \ldots, T - 1$ are as specified for the model with transfers but without sex in the previous subsection. Thus the model of transfers with sex has been constructed to be strictly comparable to the model without sex, with directly comparable notation. The allocation of resources in the model without sex is the optimal allocation for the model with sex, where the allocation will be distorted by the free-rider problem.[32]

Suppose then that the population resource allocation profile is $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^{T}$ and consider a rare mutant with profile $\{s_\tau, r_\tau\}_{\tau=1}^{T}$.[33] This mutant is subject to the growth process given by

$$\mathbf{n}^{t+1} = \mathbf{n}^t \mathbf{L},$$

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[30] Humans are actually diploid, so having two copies of each gene at each of a large number of "loci." At each locus, each offspring gets a randomly chosen gene from the mother and a randomly chosen gene from the father. Diploid sex introduces a number of complications that seem largely tangential to the present purpose.

[31] This general "non-invadability by mutants" condition could also be applied to the model of transfers without sex, and would generate the same results already found more directly for that case. It is needed here due to the game theoretic nature of the problem with sex; it was not needed without sex, since that issue is merely decision-theoretic.

[32] There is no advantage to sex in the present context. See Perry, Reny, and Robson (2008) for a discussion of the puzzle posed by the need to find the advantage of sex, of biparental sex in particular.

[33] For expositional simplicity, we restrict attention to pure population allocation profiles. However, even if there is a mixture of profiles in the population, the best reply by a rare mutant will always be a pure profile. It follows that the only possible evolutionarily stable equilibria are then pure.

where $\mathbf{n}^t$ is row-vector describing the mutant adult population at date t so that $\mathbf{n}^t = (n_1^t, \ldots, n_T^t)$. The Leslie matrix is now

$$\mathbf{L} = \begin{bmatrix} p_0\left(\frac{\bar{r}_1 + r_1}{2}\right)u_1 & p_1(s_1) & 0 & . & . & 0 \\ p_0\left(\frac{\bar{r}_2 + r_2}{2}\right)u_2 & 0 & p_2(s_2) & 0 & .. & 0 \\ \ldots & . & . & . & . & . \\ \ldots & . & . & . & . & 0 \\ p_0\left(\frac{\bar{r}_{T-1} + r_{T-1}}{2}\right)u_{T-1} & 0 & . & . & 0 & p_{T-1}(s_{T-1}) \\ p_0\left(\frac{\bar{r}_T + r_T}{2}\right)u_T & 0 & . & . & . & 0 \end{bmatrix}.$$

With the exception of the first column, this is identical to the Leslie matrix for the case without sex, and holds for the same reason—each mutant of age $\tau = 1, \ldots, T-1$ survives to age $\tau + 1$ with probability $p_\tau(s_\tau)$. Consider then the first column. Since the mutant is rare, essentially all the pairings of mutants are with nonmutants. In each such pairing, each of the $2u_\tau$ newborn offspring receive $(\bar{r}_\tau + r_\tau)/2$ but only u_τ of these offspring are also mutants. The τth term in the first column then reflects the contribution of mutant age τ parents at date t to mutant one year olds at date $t + 1$.

The limiting growth rate of the mutant type under the assumption that the mutant remains a small fraction of the population is relevant to evaluate whether the original type is robust to invasion by the mutant. The Euler-Lotka equation becomes

$$(14) \qquad 1 = \frac{p_0\left(\frac{\bar{r}_1 + r_1}{2}\right)u_1}{\lambda} + \frac{p_0\left(\frac{\bar{r}_2 + r_2}{2}\right)p_1(s_1)u_2}{\lambda^2} \cdots + \frac{p_0\left(\frac{\bar{r}_T + r_T}{2}\right)p_1(s_1)\ldots p_{T-1}(s_{T-1})u_T}{\lambda^T},$$

where λ is this limiting growth rate. The most basic biological view of preferences here, as before, is λ, but a useful economically relevant representation is the right-hand side of this equation. That is, maximizing this expression, given the *optimal* growth factor, is a necessary condition for maximizing this growth factor.

The limiting population proportions, $\mathbf{q}$, can again be taken to be $\mathbf{q} = \left(1, \frac{p_1(s_1)}{\lambda}, \ldots, \frac{p_1(s_1)\ldots p_{T-1}}{\lambda^{T-1}}\right)$, with the normalization that $q_1 = 1$. Further, the vector of reproductive values still satisfies $\mathbf{L}\mathbf{v}^T = \lambda\mathbf{v}^T$, and we set $v_1 = 1$. In this case,

$$(15)\quad v_\tau = \frac{p_0\left(\frac{\bar r_\tau + r_\tau}{2}\right)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \quad \text{for } \tau = 1,\dots,T-1,$$

$$\text{with } v_T = \frac{p_0\left(\frac{\bar r_T + r_T}{2}\right)u_T}{\lambda}.$$

These equations can again be solved by backwards recursion to obtain each v_τ as follows

$$(16)\quad v_\tau = \frac{1}{\lambda}\left\{p_0\left(\frac{\bar r_\tau + r_\tau}{2}\right)u_\tau + \frac{p_0\left(\frac{\bar r_{\tau+1} + r_{\tau+1}}{2}\right)p_\tau(s_\tau)u_{\tau+1}}{\lambda} + \cdots + p_0\left(\frac{\bar r_T + r_T}{2}\right)p_\tau(s_\tau)\dots p_{T-1}(s_{T-1})u_T/\lambda^{T-\tau}\right\},$$

for $\tau = 1,\dots,T-1$.

This expression fully spells out the reproductive value of an age $\tau = m,\dots,T-1$ adult. However, dynamic programming again provides a more compact and elegant formulation of the problem. In this formulation, individuals recursively maximize the appropriate concept of reproductive value, which we then interpret as their utility.

THEOREM 3: *The unique allocations* $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ *that satisfy the dynamic programming problem*

$$(17)\quad \max_{\substack{r_T, s_T \ge 0\\ u_T r_T + s_T = I_\tau}} \frac{u_\tau p_0\left(\frac{\bar r_\tau + r_\tau}{2}\right)}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_T, s_T \ge 0\\ u_T r_T + s_T = I_T}} v_\tau(r_\tau, s_\tau),$$

along with equation (15), *are the unique solution to the problem of maximizing the limiting growth rate of a "small" number of mutants with allocations* $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ *embedded in a population using an arbitrary allocation* $\{\bar s_\tau, \bar r_\tau\}_{\tau=m}^{T-1}$. *That is, equations* (15) *and* (17) *characterize the unique "best reply mutant allocation."*[34]

PROOF:

Dynamic programming implies that the unique solution to equations (15) and (17) maximizes the RHS of equation (14), for an arbitrary $\lambda > 0$. If $W(\lambda)$ denotes this maximized value of the RHS of equation (14), then it follows readily that (i) $W(\cdot)$ is continuously differentiable, with $W'(\lambda) < 0$, for all $\lambda > 0$; that (ii) $W(\lambda) \to \infty$, as $\lambda \to 0$; and that (iii) $W(\lambda) \to 0$, as $\lambda \to \infty$. Hence there exists a unique $\bar\lambda$ satisfying

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[34] It is not hard to show that this best reply mutant allocation profile would remain unique against an arbitrary population mixture of types with various allocation profiles. Hence, the evolutionarily stable equilibrium derived below remains unique even if mixtures are permitted.

$W(\bar{\lambda}) = 1$. This is then the maximum growth factor, since if $\lambda > \bar{\lambda}$, so that $W(\lambda) < 1$, then there is no profile of resource allocations such that λ satisfies equation (14).[35]

Under the conditions imposed on the functions $p_\tau(\cdot), \tau = 0, m, \ldots, T-1$, if these best reply allocations are interior, they must satisfy the first-order conditions

$$\frac{p_0'\left(\dfrac{\bar{r}_\tau + r_\tau}{2}\right)}{2} = p_\tau'(s_\tau)v_{\tau+1}. \tag{18}$$

Conversely, if these first-order conditions are satisfied for the mutant allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$, then these allocations characterize the mutant best reply to the allocations $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$.

We now characterize the unique evolutionarily stable outcome with transfers and sex. It is sex, of course, that implies that there are now strategic interactions, in contrast to the version of this model with no sex. If the resource allocation profile $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^{T}$ is evolutionarily stable, it is *necessary* that *one* best reply choice of mutant profile $\{s_\tau, r_\tau\}_{\tau=1}^{T}$ against the profile $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T}$ is $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T}$ itself.

Consider then the interior[36] allocation for the population given by $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ with reproductive values $\bar{v}_\tau$, uniquely characterized by

$$\frac{p_0'(\bar{r}_\tau)}{2} = p_\tau'(\bar{s}_\tau)\bar{v}_{\tau+1}, \tau = 1, \ldots, T-1, \tag{19}$$

and the recursive relationship, equation (15). Suppose that $\bar{\lambda}$ is the implied growth rate from equation (14). It is clear from equation (18) that setting $\{s_\tau, r_\tau\}_{\tau=m}^{T-1} = \{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ is then the *unique* mutant best reply to a population that chooses $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$.[37] Hence $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ is evolutionarily stable. Of course, it also follows that $\{v_\tau\}_{\tau=1}^{T} = \{\bar{v}_\tau\}_{\tau=1}^{T}$ and $\lambda = \bar{\lambda}$. Altogether, we then have

THEOREM 4: *The interior allocations* $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ *satisfying equations* (14), (15), *and* (19), *with* $\{s_\tau, r_\tau\}_{\tau=m}^{T-1} = \{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ *characterize the unique evolutionarily stable equilibrium allocation of the present model of transfers and sex.*[38]

Equation (19) implies that the unique equilibrium choice can be characterized as maximizing the "wrong" criterion from the point of view of each couple, namely, $(u_\tau p_0(\bar{r}_\tau)/2) + p_\tau(\bar{s}_\tau)v_{\tau+1}$. Thus sex leads to an attenuation of the myopic incentive to transfer resources to newborn offspring.[39] In general, with diploid sex, this is because each of these offspring have only probability $\frac{1}{2}$ of sharing a rare mutant gene present in one of the parents. With haploid sex, the probability of each offspring being a carbon copy of the mutant parent is $\frac{1}{2}$. Since the parent is unable to distinguish these

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[35] More detailed proofs of (i)–(iii) are analogous to those in the proof of Theorem 1.

[36] Equation (17) implies that the property of being a best reply to itself cannot be satisfied by any corner solution.

[37] This holds over all possible $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$, whether interior or not.

[38] This construction restricts attention to symmetric equilibria, as usual.

[39] This is "myopic" in that it holds for *given* $v_{\tau+1}$.

copies from the others, transfers cannot condition on this. This result is familiar in biology as Hamilton's rule (see Hamilton 1964, but also Bergstrom 1995).[40] From an economic point of view, children are a public good to their parents, and the undercontribution that arises is also to be expected.

Since the effect of sex is as if the true survival function for newborns $p_0(\cdot)$ were replaced by $p_0(\cdot)/2$, it straightforward to see that the result corresponding to Theorem 2 still holds:

THEOREM 5: *In the model of transfers with sex, the marginal rate of intertemporal substitution for children, those of age* $\tau = 1, \dots, m-2$ *is given by*

$$MRS_\tau = \frac{p'_\tau(I_\tau)p_{\tau+1}(I_{\tau+1})}{p'_{\tau+1}(I_{\tau+1})p_\tau(I_\tau)}.$$

Under equations (12) *and* (13), *children exhibit a rate of time discounting, from their MRS, that is positive, but decreasing with age. At the transitional age of* $\tau = m-1$, *a child has*

$$MRS_{m-1} = \frac{2\lambda p'_{m-1}(I_{m-1})v_m}{p_{m-1}(I_{m-1})p'_0(\bar{r}_m)}.$$

On the other hand, the marginal rate of intertemporal substitution for adults, those of age $\tau = m, \dots, T-1$, *is given by*

$$MRS_\tau = \frac{\lambda p'_0(\bar{r}_\tau)}{p_\tau(\bar{s}_\tau)p'_0(\bar{r}_{\tau+1})}.$$

Adults exhibit impatience, as derived from their MRS, that is at first excessive relative to the mortality and population growth, but then too small, assuming that transfers to newborns again track output, in the case with sex.

Although the myopic bias against transfers to newborns is then clear, the ultimate effect of such a bias is less clear.[41] After all, the underlying rationale for promoting your own survival instead of that of your newborns can only be future reproduction, and each of these future offspring will also only be $\frac{1}{2}$ relatives. However, it is now shown that the overall effect may still be that transfers to newborns are reduced by sex.

There is a minor complication that needs to be first addressed. This is that sex will reduce the growth rate, other things equal.[42] Indeed, the "social optimum" for maximizing λ with sex is the problem as posed without sex.

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[40] This biological motive for undercontribution arises despite all individuals being genetically identical in the equilibrium.

[41] That is, the effect of sex on the $v_{\tau+1}$ needs to be accounted for.

[42] Suppose we take this observation at face value, so that $\overline{\lambda} < \lambda^*$, say, where $\overline{\lambda}$ and λ^* are the population growth rates with and without sex, respectively. It then remains true that sex decreases impatience.

However, perhaps the most realistic comparison to make is one where both the asexual and the sexual types have reached saturation, so that the growth factor in either case is 1. Other things cannot then be equal. There must be less selection pressure with sexual reproduction than without. A plausible possibility is that this appears as a multiplicative improvement in the survival functions. This improvement will be shown to merely amplify the effects.

More specifically, suppose that the survival probabilities now have the form $B(N)\hat{p}_\tau(\cdot)$, for $\tau = 1, \dots, T-1$, where N is total population, and where the functions $\hat{p}_\tau(\cdot)$ have the same qualitative properties as $p_\tau(\cdot)$, for $\tau = 1, \dots, T-1$. The function $B(\cdot)$ is continuously differentiable and strictly decreasing. Further, we assume that $B(\cdot)$ is such that the growth factor of either the nonsexual or sexual population is greater than 1 for small enough N and less than 1 for large enough N. It follows that either population will achieve a steady state with zero growth at uniquely determined population levels, N^* and $\overline{N}$, for the nonsexual and sexual case, respectively. Since, at a population level of $\overline{N}$, the nonsexual population would grow, it follows that $N^* > \overline{N}$.

The upshot is that, if we denote the survival probabilities with sex by $\overline{p}_\tau(\cdot)$ and those without sex by $p_\tau(\cdot)$, then there exists $\beta > 1$ such that[43]

$$\overline{p}_\tau(s_\tau) = \beta p_\tau(s_\tau) \in [0, 1] \tag{20}$$

for all $s_\tau \geq 0$ and for $\tau = 1, \dots, T-1$.[44] We assume these shifts do not affect newborns, so $\overline{p}_0(r) = p_0(r)$ for all $r \geq 0$.

For the case of transfers without sex, we showed that

$$p_0'(r_\tau^*) = p_\tau'(s_\tau^*)v_{\tau+1}^*, \quad \tau = m, \dots, T-1,$$

where these optimal choices are now denoted by r_τ^* and s_τ^* and the notation v_τ^* is introduced to distinguish the two sets of shadow prices.

With sex, in contrast, we have that

$$\frac{p_0'(\overline{r}_\tau)}{2} = \beta p_\tau'(\overline{s}_\tau)\overline{v}_{\tau+1}, \tau = m, \dots, T-1.$$

In order to show that $\overline{r}_\tau < r_\tau^*$, and $\overline{s}_\tau > s_\tau^*$, it is then enough to show that $\overline{v}_{\tau+1} > v_{\tau+1}^*/2$, for $\tau = m, \dots, T-1$. Indeed, this claim holds at $\tau = T-1$ because $v_T^* = \overline{v}_T = p_0(I_T/u_T)u_T$. Adopt then the inductive hypothesis that $\overline{v}_{\tau+1} > v_{\tau+1}^*/2$ for some $\tau + 1 \in \{m+1, \dots, T-1\}$. It follows that

$$\begin{aligned}\overline{v}_\tau &= p_0(\overline{r}_\tau)u_\tau + \overline{p}_\tau(\overline{s}_\tau)\overline{v}_{\tau+1} \\ &> \frac{p_0(\overline{r}_\tau)u_\tau}{2} + \overline{p}_\tau(\overline{s}_\tau)\overline{v}_{\tau+1}.\end{aligned}$$

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[43] Of course, $\beta = B(\overline{N})/B(N^*) > 1$.

[44] The key results here concern adult impatience. These results hold under considerably weaker conditions concerning the effect of reduced selection pressure on survival. However, the multiplicative shift of survival probabilities considered here simplifies the incidental results for children.

Further, since $(\bar{r}_\tau, \bar{s}_\tau)$ maximizes this last expression,

$$\begin{aligned} \bar{v}_\tau &> \frac{p_0(r_\tau^*)u_\tau}{2} + \bar{p}_\tau(s_\tau^*)\bar{v}_{\tau+1} \\ &> \frac{p_0(r_\tau^*)u_\tau}{2} + \frac{p_\tau(s_\tau^*)v_{\tau+1}^*}{2} = \frac{v_\tau^*}{2}, \end{aligned}$$

completing the inductive proof. Hence sex unambiguously reduces all flexible transfers, those at ages $\tau = m, \ldots, T-1$, and increases the corresponding amounts used to promote own survival to the next period.

Sex thus reduces the *quality* of offspring, in that it involves reduced transfers to newborns and hence reduced survival of all of these to age one. The flip side of the coin is that adult survival rates are increased by sex relative to the model with no sex.

What is the effect of sex on impatience, as in the *MRS* ? It follows readily that there is no effect of sex on impatience of children of age $\tau = 1, \ldots, m-2$. The effect on impatience of the transitional child of age $\tau = m-1$ may be ambiguous without further assumptions, but is of limited interest.

The marginal rate of intertemporal substitution with sex is as described in Theorem 5. Denoting the resource allocation choices made with sex by $\bar{r}_\tau$ and $\bar{s}_\tau$ for $\tau = 1, \ldots, T$, the marginal rate of intertemporal substitution for adults, those of age $\tau = m, \ldots, T-1$, is

$$(21) \qquad MRS_\tau^s = \frac{\dfrac{\partial\lambda}{\partial I_\tau}}{\dfrac{\partial\lambda}{\partial I_{\tau+1}}} = \frac{p_0'(\bar{r}_\tau)}{p_0'(\bar{r}_{\tau+1})\bar{p}_\tau(\bar{s}_\tau)} = \frac{p_0'(\bar{r}_\tau)}{p_0'(\bar{r}_{\tau+1})\beta p_\tau(\bar{s}_\tau)}, \quad \tau = 1, \ldots, T-1.$$

The derivation of this expression essentially involved canceling a factor of $1/2$ from the numerator and the denominator. That is, since the individual deflates the value of resources transferred to all offspring, there is no direct effect of sex on impatience. There remains an indirect effect, however, that operates through enhanced survival of the adult from one age to the next.

Consider then the effect of sex on the impatience of adults, as in the *MRS*. Let the resource allocation choices made by adults in the model without sex be then given by r_τ^* and s_τ^*, for $\tau = m, \ldots, T$. Recall their *MRS* is then

$$(22) \qquad MRS_\tau^* = \frac{\dfrac{\partial\lambda}{\partial I_\tau}}{\dfrac{\partial\lambda}{\partial I_{\tau+1}}} = \frac{p_0'(r_\tau^*)}{p_0'(r_{\tau+1}^*)p_\tau(s_\tau^*)}, \tau = m, \ldots, T-1.$$

When we compare the expressions in equation (21) and equation (22), there is a clear effect of sex on one component of the *MRS*, the term corresponding to the pure rate of time preference. That is, we have that

$$\frac{1}{\beta p_\tau(\bar{s}_\tau)} < \frac{1}{p_\tau(s_\tau^*)}, \quad \tau = m, \ldots, T-1,$$

since sex unambiguously increases adult survival probabilities. The effect of the remaining component of *MRS*, which concerns the derivatives of the survival rates of newborns, is less clear in general, and might sometimes work in the opposite direction. In any case, one case where this remaining component has no effect is if the survival rate of newborns is linear in resources. That is, in addition to the hypotheses of Theorem 4, suppose that $p_0(r_\tau) = \alpha r_\tau$, for some $\alpha > 0$ and that adults of age $\tau = m, \ldots, T-1$ have interior optimal resource allocations, with or without sex. Now, sexual reproduction strictly decreases the *MRS* of adults of age $\tau = m, \ldots, T-1$, but it has no effect on the *MRS* of children of age $\tau = 1, \ldots, m-2$.

THEOREM 6: *In the model of transfers with sex, suppose that the same growth factor,* $\lambda = 1$, *is attained as in the model without sex by means of the shift described in equation* (20). *It follows that all adults of age* $\tau = m, \ldots, T-1$ *transfer an amount* $\bar{r}_\tau$, *say, to their newborn offspring which is strictly less than the amount transferred in the model without sex,* r_τ^*, *say. Equally, the amount such adults use for their own survival* $\bar{s}_\tau$, *say, strictly exceeds that in the model without sex,* s_τ^*, *say. As in the discussion of equation* (22), *this decreases the pure rate of time preference, and may decrease the rate of time discounting as in the MRS.*

The central message of Rogers (1994) was that sex is a key factor leading to impatience. In this model, however, this conclusion does not generally hold.

III. Conclusions

The model of transfers adopted here is one of the more tractable possible. Indeed, the model is mathematically similar to the benchmark model presented in Section I. It is not formally very different to suppose that newborns have an endogenous survival rate than it is to suppose the number of these is endogenous. In both cases, there is an endogenous number of surviving age one individuals. Indeed, since all individuals who are one or more are identical, the notion of reproductive value can be straightforwardly applied to all individuals of age one or more, despite the differences among newborns. However, the issues here are conceptually important and revealing. It is generally true that differentiated offspring require a notion of reproductive value that is not the simple expected discounted value of newborns. Further, it is only when the effect of transfers is confined to newborns that matters are as straightforward as they are here.[45]

In terms of accounting for the age profiles of empirically observed rates of time preference, it might be that the ideal model would combine intergenerational transfers, along the lines developed here, and aggregate mortality shocks, as in Robson and Samuelson (2009). The key effect of aggregate uncertainty in Robson and Samuelson (2009) was to allow the pure rate of time preference to exceed the level implied by population growth and mortality, in a observational sense. This should be superimposed on the effects found here. A further effect of aggregate uncertainty

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[45] However, a model in which transfers were made at birth, but had effects that lasted more than one period, would seem likely to remain tractable.

in Robson and Samuelson was that pure rates of time preference might decline with age and perhaps this effect would also survive in an integrated general model. Such a general model, however, would clearly push the limits of analytic feasibility.

It would also be of interest to generalize the system of transfers substantially. It is realistic, after all, to allow these transfers to be made by parents to older children as well as to newborns. Further, it is also realistic to allow transfers from parents who are beyond the age of last reproduction, not only to their children but also to their grandchildren. Such a generalization would form a useful counterpart to the investigations of Kaplan and Robson (2009). Whereas Kaplan and Robson (2009) consider a full model of social intergenerational transfers with no sex; such a generalization would involve more restricted intra-family transfers and sex.

Such more general models of transfers often become rapidly intractable. It is not hard to write an apparently simple model in which the transfer that a parent will make to a child depends not merely on the age of the parent herself, but on the age of her parent when she was born, the age of her grandparent when her parent was born, et cetera. Even if strong simplifying assumptions are made that prevent such snowballing complications, there remain a huge number of transfers that might be made in principle, not all of which will have interior solutions, so that the model will be complex to analyze.

In this light, the present model is valuable as illustrating that some central results are likely to be robust—such as the maximization of the appropriate concept of reproductive value in a dynamic programming context. More specifically, the model suggests reasons why children and younger adults may be too impatient, and older adults too patient, relative to the basic model. Finally, it provides a counterexample to any general claim that sex would generate an inappropriately high rate of time discounting. That is, although sex distorts the pure rate of time preference, it may well reduce it. Indeed, the reason why sex fails to sharply increase impatience seems likely to be a feature of many more general models.

It is important to note that, despite varying rates of time discounting, there can be no preference reversals in a frictionless model like the models used here. In the first place, time preference is tied here to age, rather than to time into the future, as is conventional in economics in general and in discussions of hyperbolic discounting in particular. Even more fundamentally, if a particular choice involving two remote ages were optimal when considered at some young age, evolved preferences would operate in models like these to ensure that such a choice would always be made in the same way even when considered at ages that were closer to the ages in question.

Another issue that arises with respect to this model is as follows. There is here a unique evolutionarily stable outcome. What then accounts for the heterogeneity in time preference actually observed? Most basically, such questions also trouble biologists. That is, why do we typically observe substantially dispersed phenotypes? One intriguing answer is provided by models of aggregate uncertainty in which the optimal genotype involves idiosyncratic randomization.[46] Another answer that

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[46] See Bergstrom (1997) for example.

might be especially relevant here is: time preference is not entirely genetically controlled but is influenced by experience as well. That is, for example, individuals who survive traumatic adverse economic outcomes might well become more risk-averse and more patient. Observed heterogeneity of preferences then simpy reflects heterogeneity in personal economic histories. However, this approach takes us even further from conventional economics and is left for future research.

The model of transfers with sex raises the following interesting issue. The game between the two parents is one where offspring survival is a public good. The undercontribution that results is not surprising in that economic light and it is in complete agreement with the biological perspective as in "Hamilton's rule." However, this prediction does not seem iron clad here. It would be more convincing in a model where the male contributes nothing to raising offspring, as is typical of most mammalian species other than our own. That is, suppose that the resources are contributed to offspring only by the female, in the complete absence of the male. Although this absence would induce her to contribute more, she would still have a marginal incentive to make transfers that was reduced on account of sex by $\frac{1}{2}$. When both parents are present and contribute, on the other hand, there is an opportunity to find ways around this undercontribution problem. These range from the biological at one end—endocrinology forging a pair bond between parents ("falling in love"), for example—to selfishly rational behavior at the other—as in the familiar ways of inducing cooperation in repeated games.

Time discounting is perhaps the key element in the debate over global warming. Most famously, Stern (2006) generates a vastly more aggressive approach to the issue than does Nordhaus (1994). This derives largely from Stern's application of a much smaller pure rate of time preference. See Nordhaus (2007) and Dasgupta (2008). Indeed, Stern's (2006) use of pure rate of time preference near zero has attracted criticism from economists as not being realistic. Although Stern's (2006) position was intended as ethical rather than realistic, a theme in the subsequent literature on global warming has been to obtain results similar to Stern's (2006) without direct appeal to low pure rates of time preference. Gollier (2010), for example, considers a model with two goods—economic and ecological. Under some assumptions, it follows that the ecological rate of discount derives from the ecological growth rate which might then fall short of the economic discount rate similarly derived from the economic growth rate. As another example, Weitzman (2010) considers a model in which the marginal product of capital is taken to be exogenous and uncertain at the start, although it is fixed in the long run. In this model, a higher coefficient of relative risk-aversion lowers the rate of discount below the risk-free rate.

How might the current analysis bear on the global warming debate? A basic issue concerns the meaning of ethical judgments. Binmore (2005), for example, argues cogently that our *actual* ethical judgments have an evolutionary basis. This position would not seem to limit the freedom of participants in the global warming debate to propose any ethical position, but rather constitutes a prediction about the guideline that is ultimately implemented. What does the present analysis predict under such a positive interpretation of ethics? It could be that the divergence between the individual and social rates of time discounting was closed as discussed in the penultimate paragraph above. This would reinforce the Nordhaus (2007) position in the sense that the actual implemented social discount rate would equal the private discount rate. More interestingly, if this divergence were not closed, the present model demonstrates a contrarian possibility—that the actual implemented social discount rate would *exceed* the individual discount rate.[47]

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[47] The underlying reason for this is simply the free-rider problem that arises with sexual reproduction. There are clearly more conventionally economic free-rider problems that arise with attempts to counter global warming that might effectively work in the reverse direction.

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