INTERNATIONAL ECONOMIC REVIEW
Vol. 46, No. 3, August 2005

COMPENSATION FOR QUALITY DIFFERENCE IN A SEARCH MODEL OF MONEY*

BY YUK-FAI FONG AND BALÁZS SZENTES[1]

Northwestern University, U.S.A.; University of Chicago, U.S.A.

We study an economy in which there is always double coincidence of wants, agents have perfect information about qualities of goods, and there are no transaction costs. The hold-up problem arises because efforts invested in improving quality prior to search may not be compensated in the market. Situations in which barter fails to motivate quality improvement are identified. With money, however, the extra effort in quality improvement will be compensated when high-quality good producers trade with agents holding both the low-quality good and money. Injection of money can induce almost all agents to produce the high-quality good.

1. INTRODUCTION

Suppose an agent has produced a commodity that is worth $100, but she chooses to consume one that costs $60. She will sell her commodity and use a part of the revenue to purchase the desired good. After these transactions, she gets to consume what she desires and keeps $40 for later purchases. The money she keeps is a compensation for the difference in values between what she produces and what she consumes. Such a compensation might not exist if there were no money.

Consider the following barter environment. Agents have produced goods of different qualities, and they are searching for trading partners. As time passes, agents who have not yet traded become more and more eager to consume. This increases their willingness to trade their goods even for goods of lower qualities. In other words, the need for consumption may cause traders with high-quality goods to face the *hold-up problem*. This is anticipated by agents before they go to the market. Hence, an agent may decide to produce the low-quality good, worrying about being held up and hoping to eventually hold up some trading partner. If the cost of producing higher-quality goods is large enough, such a disincentive for quality improvement may result in inefficiency in the economy: Even though it is still socially efficient to produce high-quality goods, only low-quality ones are produced.

How can money solve the hold-up problem and improve welfare in such an environment? Suppose some of the agents have money and their money is accepted as

* Manuscript received July 2002; revised November 2004.

[1] We are grateful to John Leahy, Ching-to Albert Ma, Robert W. Rosenthal, three anonymous referees, and especially Randall Wright for their helpful comments. Please address correspondence to: Balázs Szentes, Department of Economics, The University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A. Phone: (773) 702-9127. Fax: (773) 702-8490. E-mail: *szentes@uchicago.edu*.

a compensation for the quality difference between the high- and low-quality goods. These agents produce low-quality goods, and they offer money only if they meet traders with high-quality goods. The presence and behaviors of moneyholders have the following consequences. First, since high-quality good traders are willing to trade with moneyholders, they will leave the market earlier. This decreases the probability that a low-quality good producer meets a high-quality good producer. Second, although low-quality producers also sometimes meet moneyholders, from their point of view, a moneyholder is the same as another low-quality good producer. Both of the changes above decrease the incentive to produce low-quality goods. As a result, the quality mix of traded products can be improved.

When Kiyotaki and Wright (1989, 1991, 1993) initiated the literature on the search model of money, they illustrated how money improves welfare by eliminating the double-coincidence-of-wants problem. By modeling that goods are of different qualities and qualities are owners' private information, Williamson and Wright (1994) showed that fiat money can improve welfare by alleviating the problem of asymmetric information.[2] This study inspired several investigations of search models of money with private information of commodity qualities. Li (1995) studied commodity money and showed that a commodity with unobservable quality may be used as a medium of exchange. Trejos (1999) studied how the lemons problem affects the purchasing power of money and showed that in some equilibria uninformed buyers pay higher prices than informed buyers. Based on a model with divisible commodities and money, Berentsen and Rocheteau (2004) suggested that money has an insurance effect that induces moneyholders to produce low-quality goods, and when the information problem is not severe, money may have a negative net effect on welfare. Exploring a different role of money, Engineer and Shi (1998) showed that if barter exchanges of goods of different qualities involve "imperfect" utility transfers, then money can improve welfare, even in the absence of asymmetric information. If agents use money to trade only for the high-quality good, then frequency of inefficient barter between traders carrying goods of different qualities will be reduced and transaction cost will be saved.

Our model abstracts from all the three forms of imperfection in barter economies mentioned in the previous paragraph. In other words, (i) there is no double-coincidence-of-wants problem, (ii) agents have perfect information about the qualities of goods, and (iii) there are no transaction costs in any form in trades. In such a model, the only possible form of efficiency loss is underproduction of high-quality commodities. In this article, we focus on underproduction of quality that is due to the hold-up problem high-quality good traders encounter in the market. We let money directly compensate for the quality difference, and this proves to be a powerful policy. Whereas in the unique equilibrium outcome of the barter economy everybody is producing the low-quality good, this form of money can motivate nearly everyone to produce the high-quality good.

[2] Haegler (1997) considered a setting similar to that in Williamson and Wright (1994), except that he modeled qualities to be exogenously determined. He further confirmed this welfare-improving role of money.

In two closely related papers, Berentsen and Rocheteau (2002, 2003) also study the welfare-improving role of money in the absence of the above-mentioned imperfections in barter economies. In their papers, traders have different tastes for each other's goods. Since these differences in tastes are observable and traders negotiate before production is performed, the trader with the more preferred good has stronger bargaining power and thus produces less. Such a bargained outcome is inefficient because efficiency prescribes that this trader produces more. When money is used to transfer utility, however, the producer of the less preferred good can use money to compensate for the difference in traders' bargaining powers. As a result, money enhances gains from trade by stimulating the production of the more preferred good. The authors also showed that divisibility of money enhances the efficiency gain.

There are many important and substantial differences between the current article and Berentsen and Rocheteau (2002, 2003). First, the finding that money stimulates production in their papers is based on the assumption that traders bargain over the terms of trade before production is performed. If barter traders can sign enforceable contracts or if production can be performed instantaneously when traders meet face-to-face, this assumption is appropriate. However, there are situations in which traders have to perform productions before they go to the market, and the current article focuses on those situations. In our setup, money solves the hold-up problem faced by producers of high-quality goods, whereas in Berentsen and Rocheteau (2003) the hold-up problem is absent. Besides, in Berentsen and Rocheteau (2003) agents' individual tastes toward each other's goods are observable. By contrast, we focus on objective differences in qualities that are generally agreed upon by agents. In other words, the asymmetry of matches in Berentsen and Rocheteau is due to exogenous differences in tastes, whereas in our study it is due to endogenous production choices. Finally, Jafarey and Masters (2003) also studied a setup in which agents have different tastes for each other's goods, but their paper has a very different focus from ours.

In Section 2, we lay out the barter economy and characterize those situations where the market fails to motivate people to produce high-quality goods. In Section 3, we illustrate how introducing fiat money as a compensation for the quality difference may induce almost every agent to produce high-quality goods and enhance welfare. Section 4 concludes.

2. THE BARTER ECONOMY

2.1. *Production, Preferences, and Trading Technology.* There is a continuum of agents with a unit mass and these agents live forever. Every day, each agent chooses an effort level to produce a good of either *mediocre* or *superior* quality. The production cost of a mediocre good is normalized to 0. It costs c units of disutility to produce a superior good. Given the normalization, the appropriate interpretation of c is the cost of quality improvement. Agents never consume their own production but equally prefer goods of the same quality produced by others. In consuming other agents' goods, one superior good gives an instantaneous utility u_H and one mediocre good gives u_L, $0 < u_L < u_H$. Consuming one's own good or consuming nothing gives $-\infty$. This assumption provides a natural incentive to trade and eliminates the double-coincidence-of-wants problem. One can think of goods as being perishable or durable; either way agents must trade and consume in every period. For the ease of exposition, we assume that agents do not discount their future instantaneous utilities. The agents' preference relation $\succ$ is defined by the overtaking criterion; that is, $\{u_t - c_t\}_{t=0,1,\dots} \succ \{u'_t - c'_t\}_{t=0,1,\dots}$ if and only if $\liminf \sum_{t=0}^{T}(u_t - c_t - (u'_t - c'_t)) > 0$, where $u_t = u_H$ or u_L and $c_t = c$ or 0. For a discussion of these preferences, see Osborne and Rubinstein (1994, p. 139). Overtaking criterion is also adopted in a search model of money by Green and Zhou (2002). Our results are robust to discounting of the future; this will be discussed at the end of Section 3.

Assume that there is social gain in quality improvement, i.e., $u_H - u_L > c$. Since in our model everybody will trade every day, this assumption implies that it is always socially more efficient to have more agents producing the superior instead of the mediocre good. Finally, agents can perfectly observe the quality of each other's good before deciding whether to accept a transaction.

The timing of operation in the economy is specified as follows. There are infinitely many days. Every day, agents produce and then go to the marketplace to trade with others. The market opens in the morning. There are two rounds of trade in each day: the morning and the afternoon rounds. In each round, participants in the market are matched randomly such that agents face no aggregate uncertainty in their matches. The trading process is modeled as follows. First, both traders simultaneously display what each will offer for exchange (i.e., either nothing or the good one has produced). After seeing what each other has displayed, both traders announce simultaneously whether to accept the trade or not. The transaction is carried out if and only if both traders announce "trade." Everybody who has traded leaves the marketplace for that day.[3] There is no cost of waiting to get into the afternoon round. However, we adopt the convention that if agents expect to receive the same offer in the afternoon round of trade as in the morning, they choose to trade in the morning. Those who have not traded get into the afternoon round of trade. After goods are consumed and before the marketplace opens again the next morning, agents each produce another unit of good of either quality, according to their choices.

2.2. *Nonmonetary Equilibria.* Suppose in the production stage each agent produces the superior good with probability λ and the mediocre good with probability $(1 - \lambda)$. Then, by the Law of Large Numbers,[4] there is a proportion λ of agents producing the superior good and a proportion $(1 - \lambda)$ producing the

[3] In other words, we exclude commodity money. This assumption is made for technical convenience only. All our results remain valid qualitatively without this assumption.

[4] We are aware of the technical inadequacy of applying the Law of Large Numbers to continuum-many i.i.d. random variables; see, for example, Judd (1985), Dubey and Shapley (1994), Al-Najjar (1995), and Khan and Sun (1997). However, the reader should keep in mind that we are really interested in the case of large but finitely many traders, where we have in mind an appropriate version of the Law for which the technical problem does not arise.

mediocre good. Consider the case in which $\lambda > 0$. We call traders carrying the superior-good H traders and those carrying the mediocre-good L traders.

In the afternoon, it is always optimal for both types of traders to trade as long as the trading partner offers a good, regardless of its quality. In the morning, it is optimal to trade if and only if the trading partner offers the superior good, unless the trader surely gets the mediocre good in the afternoon. The reason is that if a trader chooses not to trade for the superior good, then she may meet another trader with the mediocre good in the afternoon. On the other hand, if a trader refuses to trade for the mediocre good, such patience always admits to a possibility of meeting a trader with the superior good in the afternoon and this trader has no better option than trading with her. It is also optimal to any strategy to go to the morning round of trade since it is costless and does not reduce the probability of meeting an H trader in the afternoon. From now onward, we assume that all agents behave this way.

Note that an L trader never gets to trade in the morning; therefore, both in the morning and in the afternoon there are measure $1 - \lambda$ of L traders. Let us look at the expected instantaneous payoffs of agents making different production decisions. First, we consider an agent who decides to produce the superior good and becomes an H trader. An H trader has a probability λ of meeting another H trader and ending up getting utility u_H in the morning. If she meets an L trader instead, which happens with probability $(1 - \lambda)$, she will not accept the trade and instead will get into the afternoon round of trade. By the afternoon, a proportion λ^2 of traders have left the market. Among the remaining $(1 - \lambda^2)$, $(1 - \lambda)$ of them are L traders and the rest, $(\lambda - \lambda^2)$ of them, are H traders. In this round, regardless of the quality of the good in hand, each trader has a $(\lambda - \lambda^2)/(1 - \lambda^2)$ probability of trading for the superior good and getting utility u_H, and a $(1 - \lambda)/(1 - \lambda^2)$ probability of trading for the mediocre good and getting utility u_L. Therefore, the expected instantaneous payoff of a superior-good producer is

$$(1) \qquad \begin{aligned} V_H(\lambda) &= \lambda u_H + (1-\lambda)\left[\frac{\lambda - \lambda^2}{1-\lambda^2}u_H + \frac{1-\lambda}{1-\lambda^2}u_L\right] - c \\ &= \lambda u_H + (1-\lambda)\left[\frac{\lambda}{1+\lambda}u_H + \frac{1}{1+\lambda}u_L\right] - c \end{aligned}$$

An L trader trades only in the afternoon where every trader will be treated identically. Therefore, the expected instantaneous payoff of an agent who produces the mediocre good and becomes an L trader is

$$(2) \qquad \begin{aligned} V_L(\lambda) &= \frac{\lambda - \lambda^2}{1-\lambda^2}u_H + \frac{1-\lambda}{1-\lambda^2}u_L \\ &= \frac{\lambda}{1+\lambda}u_H + \frac{1}{1+\lambda}u_L \end{aligned}$$

We claim the following:

Proposition 1. *(i) There always exists an equilibrium in which every agent produces the mediocre good every day. (ii) Every agent producing the superior good every day is never an equilibrium outcome. (iii) When $u_H - u_L \geq 2c$, there exists an equilibrium in which a positive fraction of agents produce the superior good. (iv) When $c < u_H - u_L < 2c$, the only equilibrium outcome is that everybody produces the mediocre good every day.*

Proof. See the Appendix.

The fact that there is more than one round of trade plays a crucial role in supporting equilibria with superior-good production. If there were only one round of trade, the extra effort in producing the superior good would never be rewarded, and it would be a dominant strategy to produce the mediocre good. In the current model, however, traders with superior goods only trade for superior goods in the morning. So, only by producing the superior good does one have a chance to obtain the superior good in the morning. Proposition 1 suggests that, if the efficiency gain from quality improvement is sufficiently high, this privilege provides enough incentive for agents to produce the superior good.

Nevertheless, when the efficient gain from quality improvement is moderate, i.e., $c < u_H - u_L < 2c$, the additional probability of obtaining the superior good in the morning is not worth the extra effort in producing the superior good, regardless of the proportion of agents who produce the superior good. The natural question is: Although the barter system fails to capture any of the potential efficiency gain from quality improvement, can introduction of fiat money into the economy help? In the next section, we provide a positive answer: Introduction of fiat money may actually implement an outcome which is *almost* Pareto optimal.

3. THE MONETARY ECONOMY

We now consider another economy that is similar to the previous one in every respect except that, in the initial period, a proportion μ of agents are provided with a unit of fiat money before they make their production decision. Henceforth, we will refer to these agents who possess money as *moneyholders*. One must be aware of two immediate consequences of introducing money. First, an agent's production decision may now depend on the amount of money she has. Second, some traders may also carry both money and goods. In the marketplace, these traders can choose to offer either nothing, just the good, just the money, or both the good and the money. Agents can choose to accumulate money, but we focus on equilibria in which agents never hold more than one unit of money.

We focus our analysis on the case when barter fails to motivate agents to produce the superior good, i.e., $c < u_H - u_L < 2c$. Our main concern here is whether there exists a monetary equilibrium that Pareto dominates the nonmonetary one. It turns out that to support a monetary equilibrium with any proportion $\lambda \in (0, 1)$ of agents producing the superior good, we just need to appropriately inject some amount of money $\mu < 1 - \lambda$. The equilibrium strategies are described of the following actions agents take every day.

A_1. A moneyholder always produces the mediocre good. She displays both the mediocre good and money in the morning and trades if and only if the trading partner displays the superior good. If she fails to trade in the morning, she goes to the afternoon round of trade. There, she hides the money and displays only the mediocre good, and agrees to trade for a good of any quality (accompanied or not accompanied by money).

A_2. An agent without money produces the superior good with probability $\lambda/(1-\mu)$ and the mediocre good with probability $1-\lambda/(1-\mu)$. Suppose she produces the superior good. In the morning, she displays the superior good and trades if and only if the trading partner either displays the superior good or both the mediocre good and money. If she does not trade in the morning, she goes to the afternoon round of trade. There, she displays the superior good and trades for a good of any quality. Suppose she produces the mediocre good instead. Then, in the morning, she displays the mediocre good and trades if and only if the trading partner displays either the superior good or both the mediocre good and money. If she does not trade in the morning, she goes to the afternoon round and displays the mediocre good and agrees to trade for a good of any quality.

Given that both actions A_1 and A_2 are taken by some players, there are three types of traders in the morning. We call the traders carrying both money and the mediocre good M traders, those carrying the mediocre good alone L traders, and those carrying the superior good H traders. In the afternoon, all traders carrying the mediocre good, with or without money, offer only the mediocre good. So, we rename all of them as L traders. Therefore, there are only L and H traders in the afternoon. The proportions of M, L, and H traders in the morning are stationary at μ, $1-\lambda-\mu$, and λ, respectively. Among them, proportion λ^2 of H traders meet each other and trade. Furthermore, $\lambda\mu$ of M traders pair up with another $\lambda\mu$ of H traders and trade. All the other traders (a proportion $1-\lambda^2-2\lambda\mu$) get into the afternoon round.

Given these strategies, we derive the expected payoffs of different agents conditioned on their production decisions. To a moneyholder who produces the mediocre good and becomes a M trader, her expected instantaneous payoff plus the expected future gain from keeping money is

$$V_M(\lambda,\mu) = \lambda u_H + (1-\lambda)\left[\frac{\lambda(1-\lambda-\mu)}{1-\lambda^2-2\lambda\mu}u_H + \frac{1-\lambda-\lambda\mu}{1-\lambda^2-2\lambda\mu}u_L + v\right] \tag{3}$$

where v denotes the expected gain from keeping money in the future. If an agent who does not have money chooses to produce the superior good and becomes an H trader, her expected instantaneous payoff net the cost of quality improvement plus the expected future gain from receiving money is

$$\begin{aligned} V_H(\lambda,\mu) &= \lambda u_H + \mu(u_L+v) \\ &\quad + (1-\lambda-\mu)\left[\frac{\lambda(1-\lambda-\mu)}{1-\lambda^2-2\lambda\mu}u_H + \frac{1-\lambda-\lambda\mu}{1-\lambda^2-2\lambda\mu}u_L\right] - c \end{aligned} \tag{4}$$

If the agent chooses to produce the mediocre good and becomes an L trader instead, her expected instantaneous payoff is

$$V_L(\lambda, \mu) = \frac{\lambda(1-\lambda-\mu)}{1-\lambda^2-2\lambda\mu}u_H + \frac{1-\lambda-\lambda\mu}{1-\lambda^2-2\lambda\mu}u_L \tag{5}$$

If the strategy profile described by (A_1, A_2) constitutes a stationary equilibrium, every agent who has no money must be indifferent between producing the superior good and the mediocre good. That is, $V_H(\lambda, \mu) = V_L(\lambda, \mu)$ must hold. Therefore, for the purpose of evaluating the payoff of one who has no money, there is no loss of generality in assuming that she commits to producing the mediocre good forever instead of randomizing. By doing so, she is an L trader forever and obtains $V_L(\lambda, \mu)$ every day.

According to A_1, a moneyholder always produces the mediocre good until she has used the money to acquire the high-quality good. Again, for the purpose of evaluating her payoff, we can assume that after getting rid of the money, she produces the mediocre good forever. Given these behaviors, the gain from holding money is realized on the day when the moneyholder, as an M trader, meets an H trader in the morning. By the Law of Large Numbers, the probability that this happens eventually is 1. The payoff of the moneyholder on that day is u_H, whereas the expected payoff of the mediocre good producer without money is $V_L(\lambda, \mu)$. Therefore, we can conclude that the value of money is

$$v = u_H - V_L(\lambda, \mu) > 0 \tag{6}$$

This also suggests that at the beginning of the first day the expected payoff of moneyholders is strictly higher than the expected payoff of other agents.

We claim the following:

PROPOSITION 2. *Suppose $c < u_H - u_L < 2c$. For any $\lambda \in (0, 1)$, there is a unique $\mu \in (0, 1-\lambda)$ such that if proportion μ of agents receive one unit of fiat money before they make their production decision on the first day, there exists a robust monetary equilibrium in which every day proportion λ of agents produce the superior good. Money is generally accepted as the compensation for the quality difference between the superior and the mediocre goods, and players follow the strategy profile described by (A_1, A_2). Furthermore, μ is decreasing in λ. All such monetary equilibrium outcomes Pareto dominate the nonmonetary equilibrium outcome, and the Pareto optimal outcome can be approximated arbitrarily closely.*

A monetary equilibrium is robust if agents strictly prefer to accept money. In order to prove Proposition 2, we first need to check that there are no incentives for the following deviations: (i) a moneyholder producing the superior good, (ii) an M trader not offering money in the morning, (iii) an M trader accepting to trade with an L or M trader or refusing to trade with an H trader in the morning, (iv) a moneyholder offering money as well as the mediocre good in the afternoon, (v) an agent without money not playing a mixed strategy in the production stage, (vi) an H trader accepting a trade with an L trader or refusing to trade with an H or M trader in the morning. Then, to prove robustness, we have to verify that an H

trader strictly prefers to trade her superior good for a mediocre good accompanied by money. The proof is provided in the Appendix.

In the proof of Proposition 1, we show that the difference between the instantaneous payoffs of an H trader and an L trader in the barter economy is

$$V_H(\lambda) - V_L(\lambda) = \frac{\lambda}{1+\lambda}(u_H - u_L) - c \tag{7}$$

When $c < u_H - u_L < 2c$, since $\lambda/(1+\lambda)$ is less than half on $[0, 1]$, this expression is always negative.

By subtracting (5) from (4), one can see that in the monetary economy, this difference becomes

$$V_H(\lambda, \mu) - V_L(\lambda, \mu) = (u_H - u_L)\frac{\lambda(1-2\mu) - \lambda^2 + \mu}{1 - \lambda^2 - 2\lambda\mu} - c \tag{8}$$

Observe that if $\mu = 0$, this equation collapses to (7), and the difference is negative. However, for any positive λ, as μ converges to $1 - \lambda$, the coefficient of $(u_H - u_L)$ converges to 1 and the difference becomes positive. Furthermore, it can be verified that the coefficient increases in μ. Hence, for any positive λ, there exists a unique $\mu \in (0, 1-\lambda)$ such that agents without money are indifferent between producing superior and mediocre goods.

Now the seemingly surprising result that a small amount of money can induce almost all agents to produce the quality goods becomes apparent. When M traders are matched with each other, they will hide their money and reenter the market in the afternoon. Hence, there are at least μ^2 traders with the low-quality good in the afternoon. On the other hand, H traders go to the afternoon market only if they meet L traders in the morning. So, in the afternoon there are at most $1 - \lambda - \mu$ traders with the high-quality good. Hence, when μ^2 is large enough compared to $1 - \lambda - \mu$ (when μ is close to $1 - \lambda$), the probability of meeting an H trader is arbitrarily small. Therefore, low-quality good production can be arbitrarily unattractive.

In the previous section, we show that the fear of being held up by L traders and the hope to hold up H traders may cause all agents to produce the low-quality good. Here, we explain how injection of fiat money motivates the production of high-quality goods by increasing the chance that H traders get compensated in the morning and decreasing the chance that L traders meet H traders in the afternoon. Take a positive proportion of superior-good production that cannot be supported as an equilibrium in the barter economy. Holding this proportion fixed, inject money into the economy so that some L traders are replaced by M traders. Notice that money is valuable because it enables moneyholders, without incurring the cost of quality improvement, to obtain the high-quality good when they meet an H trader in some future morning. Since M traders give up their money only for high-quality goods in the morning, their presence increases the payoff of H traders but not that of L traders. Moreover, it becomes less likely to meet an H trader in the afternoon for the following reasons. Some of the M traders induce some H traders to leave the market after the first round of trade and the rest of them hide their money and reenter the afternoon market. By replacing sufficiently many L traders by M traders, one can make it arbitrarily unlikely for L traders to receive the high-quality good because they can only get it in the afternoon. Besides, one can also make it arbitrarily likely for H traders to get compensated either with money or higher-quality good in the morning. This explains why replacing some of the L traders by M traders can support any level of high-quality good production as an equilibrium outcome.

In our analysis, agents lack incentives to produce high-quality goods because they know that exerting this upfront cost of quality improvement does not put them in a better bargaining position when they meet traders with low-quality goods in the market. This again stands in sharp contrast to Berentsen and Rocheteau's (2002, 2003) finding that high-quality good producers produce too little because of their strong bargaining power.

Throughout the article, we have adopted the assumption that agents do not discount the future. Nevertheless, all our results are robust to introduction of discounting. It is clear that Proposition 1 is not affected by discounting because without money agents' decisions in each period are independent of the future. The key steps in proving Proposition 2 are to show that traders holding superior goods prefer to accept money, agents prefer to use money the day after they receive it, and for every $\lambda \in (0, 1)$, there exists $\mu \in (0, 1 - \lambda)$ such that $V_H(\lambda, \mu) = V_L(\lambda, \mu)$. Here, we explain why discounting has no material impact on these steps. Since H traders' incentives to accept money is strict absent discounting, this strict preference must hold for a range of discount factors close to 1. The weak preference to use money as early as possible actually becomes strong, making the equilibrium more robust. It can be verified that if agents' discount factor is $\delta < 1$, then the value of money v will be adjusted downward from $u_H - u_L$ to $\delta\lambda(u_H - u_L)/(1 - \delta + \lambda\delta)$, lowering V_H. However, when δ is close to 1, the effect on V_H will be small and hence for $c < u_H - u_L < 2c$, except when $u_H - u_L$ is very close to c, there still exists some $\mu \in (0, 1 - \lambda)$ such that $V_H = V_L$. Since these arguments remain valid for λ arbitrarily close to 1, we can still approximate the Pareto optimal outcome.

4. CONCLUSION

In a barter economy, when indivisible commodities of different qualities are exchanged, traders carrying the high-quality good may not be properly compensated. This problem may be solved if traders carrying the high-quality good can refuse to trade for low-quality goods and continue to search. Nevertheless, as time passes, high-quality traders become more and more eager to consume and increasingly willing to trade for low-quality goods. Knowing that producing the high-quality good may not help one to obtain the good of the same quality, agents may not have enough incentives to produce high-quality goods. In a search-theoretic model, we have characterized situations in which only low-quality goods are produced in barter, and demonstrated how the introduction of fiat money improves efficiency by enhancing quality improvement.

In real life, many commodities are either by nature indivisible or for practical reasons packaged in fixed sizes. By contrast, money is printed in different denominations and its smallest denomination is as small as 1 cent. Considering an environment in which barter traders exchange production contracts, Berentsen and Rocheteau (2002) pointed out that divisible money enables traders with low-quality goods to properly compensate traders with high-quality goods and thus induces the latter to produce more.[5] By focusing on different market environments in which production precedes trades, we identify the hold-up problem as a source of efficiency loss. We believe that our results can be extended to provide a complementary explanation for why it is important to have divisible enough money when traded commodities are indivisible: The presence of divisible money as compensations for all quality differences ensures that upfront efforts in producing products of different qualities will be rewarded in the market.

APPENDIX

PROOF OF PROPOSITION 1.

(i) If every other agent produces the mediocre good, then an agent always gets the mediocre good through trade, regardless of the quality of her own good. Since producing the superior good costs more, the agent's best response is to produce the mediocre good.

(ii) Suppose every agent produces the superior good. If one of the agents deviates to produce the mediocre good, in the morning she surely meets an H trader who may refuse to trade with her. However, this H trader will surely be in the afternoon round of trade, and if there are other traders, they are all H traders. Therefore, the probability that the deviating agent obtains the superior good is 1. Thus, each player has an incentive to produce the mediocre good.

For clarity of exposition, we prove (iv) before (iii).

(iv) In order to establish this point we have to consider two different cases. Case 1: the proportion of those who produce the mediocre good is 0. Case 2: the proportion of these traders is positive.

Case 1. Consider first the possibility of having everyone except a finite number of agents producing the mediocre good. Let n denote the number of agents producing the mediocre good. The probability that an H trader meets an L trader in the morning is 0. Therefore, the payoff of a superior-good producer would be the same (i.e., $V_H = u_H - c$) as if everybody else produced the superior good. If a trader chooses to produce the mediocre good instead, then she has 0 probability of trading in the morning. In the afternoon, with probability 1 there will be $2n$ traders left, among which n are H traders. The probability for an M trader to meet an H trader will be $n/(2n-1)$. Therefore,

$$V_L = \frac{n}{2n-1}u_H + \frac{n-1}{2n-1}u_L$$

[5] Shi (1997) first extended the search model of money to one with divisible money. However, be did not analyze how divisibility of money affects welfare.

For this profile to constitute an equilibrium, it is required that $V_H = V_L$ or

$$u_H - c = \frac{n}{2n-1}u_H + \frac{n-1}{2n-1}u_L$$

Rewriting this equality, we get

$$u_H - u_L = \left(\frac{2n-1}{n-1}\right)c$$

However, this is impossible since by assumption $u_H - u_L < 2c$ and $2 \leq (2n-1)/(n-1)$.

With basically the same argument, we can argue that it also cannot be an equilibrium that there are infinitely many agents producing the superior good, but their proportion is still 0. The only difference here is that the probability of obtaining the superior good in the afternoon is now 1/2 instead of $n/(2n-1)$.

Case 2. Suppose $0 < \lambda < 1$. (Recall that λ is the proportion of people who choose to produce the superior good.) With some manipulation on (1) and (2), it can be shown that the difference in payoffs from producing the superior good and the mediocre good is

$$V_H - V_L = \frac{\lambda}{1+\lambda}(u_H - u_L) - c \tag{A.1}$$

In order to guarantee that no agent has an incentive to deviate, $V_H - V_L = 0$ must be satisfied. This implies that

$$\lambda = \frac{c}{u_H - u_L - c}$$

Since $\lambda < 1$, it follows that $c/(u_H - u_L - c) < 1$ or, equivalently, $2c < u_H - u_L$, which again contradicts the assumption that $u_H - u_L < 2c$.

(iii) Note that we only have to show that when $u_H - u_L \geq 2c$, there exists a $\lambda^* \in (0, 1)$ such that $V_H - V_L = 0$. From (A.1), we can see that if λ is close to 0, $V_H - V_L$ is negative. If λ is close to 1, then $\lambda/(1+\lambda)$ is close to 1/2. Since $u_H - u_L > 2c$, this means that $V_H - V_L$ is positive. Since $V_H - V_L$ is a continuous function of λ, we can conclude that there exists a $\lambda^* \in (0, 1)$ such that $V_H - V_L = 0$. ■

PROOF OF PROPOSITION 2.

(i) It is optimal for moneyholders to produce the mediocre good. If a moneyholder produces the superior good, it is easy to verify that she has no incentive to offer money in any round. If she postpones the opportunity to use the money for a finite number of days, then she still achieves the same benefit of holding money, v, because there is no discounting. If she postpones forever, then it is a waste of the money.

(ii) In the morning, it is optimal for an M trader to offer both money and the mediocre good. By hiding money, an M trader gets $V_L(\lambda, \mu)$ instead of $V_M(\lambda, \mu)$ in that period. Again, hiding the money in the morning for a finite number of days does not affect the utility, but hiding it forever wastes the money.

(iii) In the morning, it is optimal for an M trader to trade only with an H trader. If an M trader trades with an L or M trader, then she loses the positive probability of meeting an H trader in the afternoon, which provides her with higher instantaneous utility. It is always better not to trade in the morning but enter the afternoon round of trade with money hidden. By doing so, she still keeps the money but gets $V_L(\lambda, \mu) > u_L$ instead.

(iv) In the afternoon, it is optimal for a moneyholder to offer only the mediocre good. Since the trading partner always accepts a trade as long as she is offered something that provides positive utility, there is no need to offer money.

(v) Agents without money produce the superior good with probability $\lambda/(1-\mu)$. We show that for each $\lambda \in (0, 1)$ there exists a μ such that

$$V_H(\lambda, \mu) = V_L(\lambda, \mu) \tag{A.2}$$

Define $f(\lambda, \mu) \equiv V_H(\lambda, \mu) - V_L(\lambda, \mu) = \lambda(u_H - V_L) + \mu(u_L + u_H - 2V_L) - c$. We show that for all $\lambda \in (0, 1)$, there exists a unique corresponding $\mu(\lambda)$ such that $f(\lambda, \mu(\lambda)) = 0$. We are going to argue that if μ is 0, $f(\lambda, \mu)$ is negative, and if μ is $1 - \lambda$, then $f(\lambda, \mu)$ is positive. Since f is continuous on $\{(x, y) \in \mathbb{R}^2 : x \in (0, 1), y \in [0, 1 - x]\}$, there must exist a $\mu \in (0, 1 - \lambda)$ for which $f(\lambda, \mu) = 0$. In addition, we show that $f(\lambda, \mu)$ is strictly increasing in μ; hence, $\mu(\lambda)$ must be unique. Using (5),

$$f(\lambda, \mu) = (u_H - u_L)\frac{\lambda(1-2\mu) - \lambda^2 + \mu}{1 - \lambda^2 - 2\lambda\mu} - c \tag{A.3}$$

Therefore,

$$f(\lambda, 0) = (u_H - u_L)\frac{\lambda}{1+\lambda} - c$$

In the proof of Proposition 1, part (iii), Case 2, we have shown that this expression is indeed negative. Furthermore,

$$\lim_{\mu \to 1-\lambda} \frac{\lambda(1-2\mu) - \lambda^2 + \mu}{1 - \lambda^2 - 2\lambda\mu} = \left.\frac{\lambda(1-2\mu) - \lambda^2 + \mu}{1 - \lambda^2 - 2\lambda\mu}\right]_{\mu=1-\lambda} = 1$$

Hence,

$$f(\lambda, 1-\lambda) = (u_H - u_L) - c$$

which is positive by assumption.

We have shown that $f(\lambda, \mu) < 0$ when μ is 0 and $f(\lambda, \mu) > 0$ when μ is $1-\lambda$. Since

$$\frac{\partial f(\lambda, \mu)}{\partial \mu} = (u_H - u_L)\frac{(\lambda-1)^2}{(-1+\lambda^2+2\lambda\mu)^2} > 0 \quad \text{if } \mu < 1-\lambda$$

we can conclude that $f(\lambda, \mu)$ is strictly increasing in μ. Hence, for each $\lambda \in (0, 1)$, there exists a unique $\mu \in (0, 1-\lambda)$ such that $f(\lambda, \mu) = 0$.

(vi) In the morning, an H trader strictly prefers to trade with an M trader or an H trader but refuses to trade with an L trader. To ensure that an H trader strictly prefers to trade with an M trader, it must be that

$$u_L + v > V_L$$

Plugging in v from (6) and V_L from (5), this inequality can be rewritten as

$$u_L + u_H > 2\frac{\lambda(1-\lambda-\mu)}{1-\lambda^2-2\lambda\mu}u_H + 2\frac{1-\lambda-\lambda\mu}{1-\lambda^2-2\lambda\mu}u_L \tag{A.4}$$

Observe that on both sides of the inequality, the weights of u_L and u_H add up to 2. The inequality holds because $\lambda(1-\lambda-\mu) < (1-\lambda-\lambda\mu)$ and $u_L < u_H$. Since $u_H > V_L$, she is also willing to trade with an H trader. Since $u_L < V_L$, she prefers to go to the afternoon round instead of trading with an L trader.

(vii) Finally, we show that $\mu(\lambda)$ is decreasing. Since $f(\lambda, \mu)$ is strictly increasing in μ (see (v)), it is enough to show that $f(\lambda, \mu)$ is strictly increasing in λ. But

$$\frac{\partial f(\lambda, \mu)}{\partial \lambda} = (u_H - u_L)\frac{1+\lambda^2+2\lambda\mu-2\mu-2\lambda+2\mu^2}{(-1+\lambda^2+2\lambda\mu)^2}$$

It is easy to show that at $\lambda = 0$ this expression is positive and the numerator (as a function of λ) has only complex roots. Therefore, $\partial f(\lambda, \mu)/\partial\lambda$ must be positive and $f(\cdot, \mu)$ must be increasing on $(0, 1-\mu]$.

Therefore, we have shown that there exist monetary equilibria in which a positive proportion of the agents produce superior goods. The robustness of the monetary equilibrium follows from the strict inequality (A.4). According to Proposition 1, in every nonmonetary equilibrium all agents produce mediocre goods when $c \le u_H - u_L \le 2c$. Therefore, these monetary equilibrium outcomes Pareto dominate the nonmonetary equilibrium outcome. Furthermore, since λ can be arbitrarily close to 1, the Pareto optimal outcome in which every agent consumes a superior good can be approximated arbitrarily closely. ■

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