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The price of advice

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We develop a model of consulting (advising) where the role of the consultant is to reveal signals to her client that refine the client's original private estimate of the profitability of a project. Although the client can perfectly observe and evaluate these signals, the consultant may only be able to do the same imperfectly, or not at all. This captures the idea that the expert may not fully understand the impact of her advice on the client. We characterize the optimal contract between the consultant and her client. It is a menu consisting of pairs of transfers specifying payments between the two parties in case the project is undertaken by the client and in case it is not. The main result of the article is that in the optimal mechanism, the consultant obtains the same profit as though she could perfectly observe and evaluate the impact of the signals whose release she controls on the client's profit estimate.

1. Introduction

■ In many real-world advisor-client relationships, the degree to which the expert understands the impact of her information on the client's objectives lies between two extremes: complete understanding and total ignorance. For example, a competent personal tax advisor is expected to know not only the tax law but also how certain regulations affect her client's actual tax obligations. On the other hand, a computer scientist who is invited to explain cryptographic techniques to counter-terrorism officials may be intentionally kept in the dark about aspects of the project for which her advice is needed. In this article, we study models where the situation is anywhere between these two extremes, and the client's action (what he does with the advice) is contractible. We try to understand how consultants "create value" and characterize optimal contracts between them and their clients.

We propose a model where a consultant (she) is able to reveal signals to her client (him) that refine the client's original private estimate regarding the profitability of a project. We assume

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that the client can perfectly observe and evaluate these additional signals; however, the consultant may only be able to do the same imperfectly. This captures the idea that the expert may not fully understand the impact of her advice on the client. In an extreme case, she may not even know *a priori* whether her advice made the project look more or less profitable to the client. However, the parties' actions—in particular, whether or not the project is undertaken—are observable and contractible, hence the consultant can make inferences from the client's choices. The central question is how much information the consultant should disclose (with or without observing), and how she can reduce the client's rents stemming from his ability to see and evaluate her signals.[1]

We characterize the optimal contract between the consultant and the client. It can be represented by a menu that consists of pairs of transfers specifying payments between the two parties contingent on whether or not the project is undertaken by the client. If the client chooses an item from the menu, the consultant agrees to release to him whatever information she has, and the transfers take place according to the client's action.[2] In the optimal menu, there may be items where the client pays a fee to the consultant upon undertaking the project, as well as items where the consultant pays the client if the project is carried out. The client's choice among the different pairs of transfers depends on how optimistic or pessimistic he is regarding the profitability of the project prior to listening to the advice of the consultant. Intuitively, the client is willing to pay for the consultant's advice because it may induce him to take an action different from the one he was planning to take. Indeed, in certain special cases, the client pays the consultant a positive fee exactly when her advice changes the client's mind as to whether or not to undertake the project. If the client is either very optimistic or very pessimistic regarding the success of the project, he chooses not to contract with the consultant.

The main result of the article is that even if the consultant cannot perfectly (or at all) evaluate the "new information" that she discloses, she can design a contract in which she obtains the same profit as though she could. In other words, in the optimal contract, the client enjoys information rents only for the information he already has prior to meeting the consultant. The client does not get any rents from the information whose release is controlled by the consultant, even if that information becomes his (the client's) private information when released. We also show that the optimal contract does entail inefficiencies, that is, the first-best is not achieved.

□ **Motivating examples, applications.** Suppose that a large software company (e.g., Google) is looking for a senior engineer to oversee the development of a new, secret product. They bring in a headhunter to evaluate potential candidates. The headhunter (in our model, the consultant) has superior information about the skills of the candidates, whereas only the company (the client) knows exactly what project the new chief engineer would be in charge of. (The project and the job description are kept secret because Google does not want to risk a competitor preempting their product launch.) Therefore, when describing a candidate, the consultant may not exactly know whether a specific characteristic makes the candidate more or less attractive for the client, only the client does. However, the client's hiring decision is observable and contractible. In this setup, we look for the headhunter's optimal contract, which includes the rules of information disclosure and payments contingent on the client's action.

Another situation that fits our model is where the client is a potential investor in a firm's shares. He is uncertain of various characteristics of the asset, such as the covariance of its return with other stocks' returns. The consultant is a finance expert who has access to this type of information. She does not know the client's willingness to pay for the stock and, in certain situations, she may not even know precisely how her information would *change* the client's

[1] The consultant may hesitate to reveal to the client everything she can because information disclosure is costly for her. If the client's action was not observable, then in order to reduce the payment to the consultant, he would always claim *ex post* that the consultant's information was not helpful.

[2] A contract, in general, could be more complicated (for example, it could involve lotteries); however, we show that the optimal contract has this simple form.

valuation. This can be the case if the client cares about the covariance of the stock's return with the return of his own portfolio whose composition is also his private information.[3] The goal is to design the optimal contract for the consultant when the client's action (whether or not he buys the stock) is contractible, but the effect of the consultant's information on the client's valuation is not.

Yet another example is where the consultant is a real estate broker and the client is a prospective home buyer. The buyer's preferences over houses are his private information. The broker has detailed information about the amenities that various homes offer, but she does not know what the buyer is looking for. For example, is a neighborhood with an active night life a plus, or a minus? The buyer may not be able or *inclined* to disclose his preferences—after all, the broker is a self-interested economic agent, and she would use the information gleaned from the buyer to her own advantage. (If the broker becomes fully informed about the buyer's preferences, then she can deliberately show expensive homes with amenities that the buyer likes in order to increase her commission set as a fixed percentage of the price.)

More broadly, it is a widely held belief that the role of strategy and management consultants is to help uncover their clients' own ideas so that the clients can realize what they are capable of.[4] Consultants often only talk about the correct general criteria to be used in decision making (what types of tradeoffs to consider, common fallacies, etc.), instead of the particularities of the client's decision problem. By discussing general ideas, industry trends, or similar cases, they provide useful information as the client's private knowledge regarding his project becomes more nuanced. Nevertheless, the consultant may only imperfectly (or never) learn exactly what effect her advice has had on the client's objective function.[5] In many cases, it is conceivable that only the client's actions are observable and contractible. We characterize the contract that maximizes the consultant's profit. This reflects the assumption that the consultant has a (local) monopoly on the type of information sought by the client.

Our results imply that in all the applications mentioned above, the consultant should offer a menu of contingent-fee contracts where her remuneration depends on the client's action. We show that by offering such a contract, the consultant can extract as much profit as if she could perfectly observe the actual effect of her advice on the client's objective function. Our main result implies that it is irrelevant how well the consultant understands the impact of her advice on the client—as long as she controls the disclosure of information, she appropriates its rents.

In many real-world examples of professional advice, such as real estate, law, management, or IT consulting, we observe fees that are contingent on the client's action. In mergers, for example, it is customary for the consultant of the buyer to demand a "success fee" due upon the completion of the deal. Because the acquisition of the target is ultimately the client's decision, we may interpret such success fees as action-contingent transfers. Our model shows that this may be the outcome of an optimal contract between the consultant and her client. In this contract, the consultant discloses as much information as she has even though she is unaware of its impact on the client's preferences, because she captures all the additional rents arising from the information whose release she controls.

□ **Related literature.** The literature on information transmission between experts and client-customers (see Milgrom and Roberts, 1986, Pitchik and Schotter, 1987, Wolinsky, 1993, Emons, 1997; and references therein) often treats the expert's information disclosure as *cheap talk* (à la Crawford and Sobel, 1982). The advisor has unverifiable information, and the question is how

[3] The composition of the client's portfolio may be difficult to communicate, or the client may feel uneasy about disclosing the details of his financial situation when all he needs is advice on the value of a particular stock.

[4] Accenture (a consultancy) advertises its "ability to act as a catalyst" to "bring [clients'] ideas to life" (Accenture, 2002).

[5] It may be the case that the client intentionally restricts the consultant's ability to evaluate the client's options, and this is why only the client can interpret the consultant's advice. For example, the client may fear that the consultant would use her knowledge of the client's problem to advise his competitors.

precisely she can reveal it if the interests of the parties are not perfectly aligned and the client's actions are not contractible. Our model does not have much in common with cheap-talk models, as in our setup it is the client, not the expert, who can perfectly observe the disclosed signals; moreover, the client's action is contractible.

There are other, more related models of advice in the literature concerning the attorney-client relationship. This line of research is motivated mostly by the observation that attorneys are paid contingent fees, that is, payments that substantially differ depending on the success or failure of the client's case. The literature (see Dana and Spier, 1993 and references therein) offers several types of economic explanations for such contracts, among them risk sharing, liquidity-constrained clients, and moral-hazard problems associated with the attorney. Contingent-fee contracts may also be optimal when there is asymmetric information between the attorney and the client. In the model of Scotchmer and Rubinfeld (1990), the attorney has private information about her own ability, whereas the client is better informed regarding the merits of the case. In Dana and Spier (1993), the attorney obtains superior information regarding the merits of the client's case after having offered the client a contract. In both models, contingent-fee contracts arise in equilibrium.

Our setup is different from these in that we consider an advisor (in this application, the attorney) who can make her client better informed about the client's project (the legal case), without necessarily becoming better informed herself.[6] We derive the contract that is optimal for the advisor in this setting. Interestingly, the optimal contract in our model resembles the ones seen in the literature cited above; the main differences are that in our case, it is a *menu* of contingent fees, and that the transfers are conditional on the client's observable *action* (e.g., whether he decides to pursue the case), not the outcome.

In a related article (Eső and Szentes, 2007), we analyze the auction design problem where a monopolist can disclose, without observing, private signals to the buyers that refine their initial private valuation estimates for the object being sold. That article characterizes the revenue-maximizing selling mechanism and shows that in the optimal mechanism, the seller discloses all available signals (which only the buyers can observe) and attains the same revenue as though she could directly observe the realizations of these signals. This result is similar to the one we obtain in the present article, where the consultant obtains the same profit as if she could observe the effect of her signal on the client's valuation for the project. The problem is very different here, however, because it is not the seller, but a third party, who controls information relevant for the buyer (here, the client).

In Baron and Besanko (1984), Riordan and Sappington (1987), and Courty and Li (2000), a principal and an agent are contracting over two periods. Independently of the contract, the agent learns payoff-relevant private information in both periods. These articles analyze the optimal two-stage revelation mechanism where the contract is signed in the first period, when the agent knows only his first-period type. In contrast, in the present article, it is a *third party* (the consultant), not the principal, who can release additional information to the agent, and the consultant can *decide* whether or not to release her information.

Our model is technically a principal-agent model where the value of the agent's outside option depends on his type.[7] Principal-agent models with type-dependent outside options have been studied in the literature by Lewis and Sappington (1989), Klibanoff and Morduch (1995), Maggi and Rodriguez (1995), and most generally by Jullien (2000). Our derivation of the optimal contract in the case where the consultant can perfectly observe the impact of her information on the client's valuation, discussed in Section 3, relies on insights that are familiar from this

[6] This may be a good assumption if the client has pertinent information about his own case that he does not share with his attorney. In an extreme case, the client could simply ask the attorney to clarify the law for him so that he can make a better decision about whether or not to pursue the case, and the attorney remains forever ignorant about her client's objectives.

[7] The client can undertake the project without asking the consultant for advice. If his original value estimate (type) is below the project's cost, then the client's outside option is worth zero. If his estimate exceeds the project's cost, then the client's outside option is the project's net profit, which is increasing in his type.

literature.[8] Naturally, the question whether the consultant could gain by directly observing the signals that she controls does not arise in this literature, because that is a question very specific to our actual model.

The article is structured as follows. In the next section, we outline the model and introduce the necessary notation. In Section 3, we derive the optimal contract for the consultant and show that it yields the same expected profit no matter whether or not the consultant can observe the signal that she releases. Section 4 concludes.

2. The model

■ There are two risk-neutral agents in the model: a consultant (she) and her client (he). The client can undertake a project at a cost r, where $r \in \mathbb{R}$ is commonly known. The project generates a stochastic *ex post* monetary benefit, $V = v + s$, where v is the client's estimate of V, and s is an error term. Note that the additive structure is not an assumption, rather, $s \equiv V - v$ is the definition of the error term.

The variable v is the client's private information. The consultant can recover and disclose the signal s to the client at a cost K. We assume that the consultant does not observe the signal s, but only a signal z which may be correlated with s. (The variable z is nevertheless independent of v.) It is important to understand that this assumption—that the sender of s does not perfectly observe s whereas the receiver does—is just the way we *model* the situation where the consultant may be unaware of the precise effect of her information on the client's value estimate.

In the two extreme cases, z is either perfectly correlated with s, $z = s$, or z is independent of (uninformative about) s. Our main result is that the consultant's payoff from the optimal contract does not depend on the information content of z. That is, even if the signal z is only imperfectly (or not at all) correlated with s, the consultant can design a contract in which she obtains the same profit as though $z = s$.

We assume that v is drawn from a distribution F on the unit interval[9] with a positive density f that is twice differentiable and log-concave (i.e., $d^2 \ln f(v)/dv^2 \leq 0$). Log concavity is an important, though standard, assumption in the literature on contracting with incomplete information. It implies, among other things, that the distribution satisfies certain monotone hazard rate conditions. In particular, for all $b \in [0, 1]$, $(b - F)/f$ is weakly decreasing.[10] Many widely used density functions satisfy log concavity (see Bagnoli and Bergstrom, 1989).

The other component of the client's *ex post* valuation, s, is drawn from a distribution G with full support on $(-\infty, +\infty)$.[11] We assume that s and v are independently distributed and $E[s] = 0$. In words, the client's unbiased estimator of his valuation is v (with an independent error term), and he has no other private information, for example, regarding the precision of this estimator. It is without any loss of generality to normalize the expectation of s to zero; the independence assumption is made for the sake of conceptual clarity. In the discussion paper version of this article (Eső and Szentes, 2004), we show under what conditions and how the analysis can be generalized when the error term ($s \equiv V - v$) is correlated with v.

In our model, the only role of the consultant is that she can refine the client's original value estimate by disclosing, without perfectly observing, s.[12] We assume that the consultant's

[8] More generally, in Section 3 we use well-known techniques of contract theory and mechanism design. For a textbook reference, see Salanié (1998).

[9] The normalization that the project's expected gross profit, v, falls between 0 and 1 is innocuous, as r can be positive, negative, or zero. All that this assumption implies is that the project's expected profit is bounded.

[10] This result is due to Prékopa (1971). For references, see also Fudenberg and Tirole (1991) and Jullien (2000).

[11] Intuitively, the full-support assumption ensures that no realization of V (small or large) can be excluded given a particular estimate v. This assumption is made solely for ease of exposition. All our results go through (with more cumbersome notation) if the support of the distribution of s is not the whole real line.

[12] The model could be easily modified to allow for the consultant's advice to have a fixed, positive effect on the client's *ex post* valuation as well. This would not change the results.

action—whether or not she provides advice—is contractible (i.e., there is no moral hazard on her part), and that she cannot garble s (i.e., change its value if she discloses it).[13]

What is the social "value" of the consultant's services? Intuitively, the closer the client's profitability estimate to the cost of the project, the more valuable it is to learn precisely the project's true gross value. Formally, the calculation is the following. If s is observed, then it is socially valuable for the client to undertake the project if and only if $v + s \geq r$, and the social surplus generated this way is

$$\int_{r-v}^{\infty} (v + s - r)\, dG(s). \tag{1}$$

If s is not observed and $v \leq r$, then the project should not be undertaken and the social surplus is zero. Therefore, if $v \leq r$, then the social value of learning s is exactly (1). If $v \geq r$, then the project should be undertaken without knowing s, which yields a social surplus of $v - r$. If the consultant discloses s, the social surplus becomes (1), hence the gain from learning s is $\int_{r-v}^{\infty}(v + s - r)\, dG(s) - (v - r)$. Therefore, the gross social value of the consultant's information is

$$w(v) = \int_{r-v}^{\infty} (v + s - r)\, dG(s) - \max\{v - r, 0\}. \tag{2}$$

This function is strictly increasing for $v < r$ and strictly decreasing for $v > r$. We assume that the release of information costs *less* to the consultant than the gross social value of her information no matter what the client's type is, that is, $w(0) \geq K$ and $w(1) \geq K$. This assumption means that it is always socially desirable for the consultant to release her information to the client. However, the consultant does not always do so in the optimal (expected revenue-maximizing) contract, as we show in Section 3.

Our goal is to characterize the contract that maximizes the consultant's profit. The contract is offered at the interim stage when the client already knows v. Its terms cannot depend directly on the realization of s, only on the realization of z. However, whether or not the project is undertaken is contractible. In particular, the transfer that the client pays for the disclosure of s may depend on whether or not he executes the project.[14] We shall not give an exact characterization of the available contracts. Instead, we derive the optimal contract in the case where $z = s$. Then, we show that the consultant can achieve the same profit even if z and s are independent or imperfectly correlated.

The simplest situation that corresponds to the model's formalism is where the client is the management of a firm contemplating the acquisition of another firm. The takeover price is r (commonly known); the client's initial estimate about the target's value is v (privately known). The consultant is an expert on mergers. She can help (at a cost of $K \geq 0$) the client find out the value of the target firm without her actually learning anything, including the change in the client's valuation for the target. The consultant designs the contract under which she provides advice. The client's action whether or not to proceed with the merger is observable and contractible. The consultant's goal is to maximize her expected profit.

In our setup, the consultant acts as a monopolist when offering a contract to the client. This is an abstraction of the fact that advice is a differentiated product and consultants enjoy limited market power. In reality, contracts are negotiated between consultants and clients, and usually neither party has a complete advantage in the process. However, a situation where the bargaining power is shared between the consultant and the client could be modelled such that at time 0, a lottery determines who has the right to make the offer. If the client gets to offer a contract, the optimal contract is simple: he asks the consultant to give him advice in exchange for a transfer

[13] Garbling the advice is not a practical possibility for real-world consultants, and it would trivialize the theoretical analysis.

[14] The client may randomize between undertaking and not undertaking the project; similarly, the consultant may also offer to use a lottery to decide whether or not to disclose s to the client.

of K. In the rest of the article, we focus on the case where the bargaining power is delegated to the consultant, which should be thought of as a subgame of the larger game describing the negotiation process.

The willingness-to-pay function in (2), together with the distribution of v, could be used to compute the monopoly price for the consultant's services, that is, an optimal flat fee that she could charge for disclosing (with or without observing) her information. However, the purpose of this article is to investigate what (how much more) can be done when the client's action as to whether or not he undertakes the project is contractible. We will see that flat-fee contracts are not optimal.

3. The consultant's optimal contract

■ First we characterize the optimal mechanism under the assumption that the consultant, after having committed to a mechanism, *can perfectly observe* the realization of s at a cost K, that is, $z = s$. The solution to this relaxed problem clearly provides an upper bound on her profit in the case where z is only imperfectly correlated with s.[15] Then we show that the same outcome can be implemented even if the consultant cannot observe s at all, that is, z is independent of s. This provides a lower bound on her profit in the case where z and s are correlated. Because the lower and upper bounds on the consultant's payoff are the same, we can conclude that it does not depend on z. In other words, although the client still enjoys information rents from his type, all his rents from observing the consultant-controlled signal is appropriated by the consultant. This is the main result of the article. We then argue that the incentive constraints are stronger in the case where z and s are independent than in the case where $z = s$, that is, the two problems are generally not equivalent.

□ **The consultant can observe s.** Suppose first that the consultant can pay K (the cost of providing advice) in order to observe the realization of s.[16] She offers the contract to the client at the interim stage when she has not seen s but the client has already observed v. By the revelation principle, all contracts can be represented by truthful direct revelation mechanisms. Such mechanisms consist of four functions: $t : [0, 1] \to \mathbb{R}$, $a, \bar{x} : [0, 1] \to [0, 1]$, and $x : [0, 1] \times (-\infty, \infty) \to [0, 1]$ with the following interpretation.

$t(v)$ is the expected transfer from client type v to the consultant.
$a(v)$ is the probability that the consultant checks the realization of s.
$x(v, s)$ is the probability that the client undertakes the project when the consultant observes the realization of s.
$\bar{x}(v)$ is the probability that the client undertakes the project when the consultant does not observe s.

The consultant offers the contract $\{a, x, \bar{x}, t\}$, and the client can either accept or reject it. If he rejects it, then his payoff is $\max\{v - r, 0\}$. If he accepts it, then he reports a type, v, and pays the consultant $t(v)$. The consultant checks the realization of s with probability $a(v)$, and directs the client undertake the project with probabilities $x(v, s)$ and $\bar{x}(v)$, in case she did and did not observe the value of s, respectively. The mechanism satisfies the participation constraint if it induces all types of the client to accept the contract; it is incentive compatible if it induces the client to report v truthfully.

This is a principal-agent model where the consultant is the principal and the client is the agent. There are two "twists" relative to a standard problem: (i) the principal can observe, at a

[15] This is so because if the consultant can observe s, she can always generate a random variable z that is arbitrarily correlated with s, and commit to make the contract depend on z but not on s.

[16] For simplicity, assume that the client also observes s when the consultant decides to uncover it, although this does not matter as long as the consultant offers the contract.

cost, a valuable signal, s; and (ii) the value of the agent's outside option (his reservation utility) is max $\{v - r, 0\}$, which is therefore type dependent.[17]

Let X be the expected probability that type v undertakes the project:

$$X(v) = a(v) \int x(v, s)\, dG(s) + (1 - a(v))\bar{x}(v). \tag{3}$$

Denote the deviation payoff for type v reporting v' by

$$\pi(v, v') = a(v') \int x(v', s)(v + s - r)\, dG(s) + (1 - a(v'))\bar{x}(v')(v - r) - t(v').$$

Incentive compatibility requires $\pi(v, v') \leq \pi(v, v)$ for all v, v', whereas participation requires $\max\{v - r, 0\} \leq \pi(v, v)$ for all v.

Define $\Pi(v) = \pi(v, v)$, the client's payoff when he announces v truthfully. A useful technique introduced by Mirrlees (1971) is to replace the transfer function (t) with the client's indirect profit function (Π) in the definition of a mechanism. In what follows, we refer to a contract (or direct mechanism) by the tuple $\{a, x, \bar{x}, \Pi\}$. The following result characterizes all incentive-compatible mechanisms.

Lemma 1. A direct mechanism is incentive compatible, if and only if,

$$\Pi(v') - \Pi(v) = \int_v^{v'} X(z)\, dz, \tag{4}$$

and X is weakly increasing.

Proof. Standard and omitted (see, e.g., Salanié, 1998).

The functions a, x, and $\bar{x}$ determine the total surplus generated by the contracting parties and, by (4), the client's profit up to a constant. The consultant's payoff equals the social surplus less the client's profit,

$$W(v) = a(v) \int x(v, s)(v + s - r)\, dG(s) + (1 - a(v))\bar{x}(v)(v - r) - \Pi(v). \tag{5}$$

Lemma 1 allows us to reinterpret the consultant's mechanism design problem in the usual way as follows. She is trying to set the "decision rule" (the functions a, x, and $\bar{x}$, which determine X) and the "boundary values" for Π (e.g., $\Pi(0)$ or $\Pi(1)$) such that the *ex ante* expectation of her surplus, $\overline{W} = \int_0^1 W(v)\, dF(v)$, is maximized, subject to the incentive compatibility constraints characterized by the lemma, and the participation constraint, $\Pi(v) \geq \max\{v - r, 0\}$.

The slope of Π is between zero and one by (4), whereas the slope of the client's participation constraint, $\max\{v - r, 0\}$, is zero for $v < r$ and one for $v > r$. Therefore, it is clear that in the optimal contract, the participation constraint binds either for $v = 0$ or $v = 1$, or both, as illustrated in Figures 1–3 below. It is also clear that the client's participation constraint may bind only for types *outside* some open interval.

In what follows, we provide a heuristic derivation for the optimal contract in each of the three cases seen in the figures. We also explain which of the three cases applies for which parameter values. We collect the results in Proposition 1, whose precise proof is completed in the Appendix.

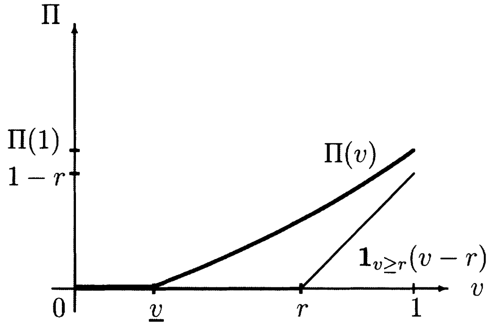
Case 1. Suppose that in the optimal contract, the lowest type's participation constraint binds but the highest type's does not, as in Figure 1. An immediate consequence of Lemma 1 (obtained by integration by parts) is that the client's *ex ante* expected payoff can be expressed as

$$\int_0^1 \Pi(v)\, dF(v) = \Pi(0) + \int_0^1 (1 - F(v))X(v)\, dv.$$

[17] Static models where the agent's outside option is type dependent are solved by Lewis and Sappington (1989), Maggi and Rodriguez (1995), Klibanoff and Morduch (1995), and Jullien (2000). Our solution for the relaxed problem (where $z = s$) relies on the techniques developed in these and other articles on Bayesian mechanism design.

FIGURE 1

THE PARTICIPATION CONSTRAINT DOES NOT BIND AT $v = 1$



Combining this with equations (3) and (5) yields for the consultant's expected payoff,

$$\overline{W} = \int_0^1 \left\{ a(v)\left[\int \left(v + s - r - \frac{1 - F(v)}{f(v)} \right) x(v,s)\, dG(s) - K \right] \right.$$
$$\left. + (1 - a(v))\left(v - r - \frac{1 - F(v)}{f(v)} \right) \bar{x}(v) \right\} dF(v) - \Pi(0). \qquad (6)$$

$\overline{W} + \Pi(0)$ can be interpreted as the expected "virtual surplus." For client type v, the virtual surplus is $(v + s - r - (1 - F(v))/f(v))\, x(v, s) - K$ when s is observed, and $(v - r - (1 - F(v))/f(v))\bar{x}(v)$ when s is not observed.

We want to maximize (6) by choosing a, x, $\bar{x}$, and $\Pi(0)$. We set $\Pi(0) = 0$. First, notice that for any function a, $\overline{W}$ is pointwise maximized if $x(v, s)$ equals one whenever $v + s - r \geq (1 - F(v))/f(v)$ and zero otherwise, and if $\bar{x}(v)$ equals one whenever $v - r \geq (1 - F(v))/f(v)$ and zero otherwise. This means that the client undertakes the project whenever the expected net value of the project ($v + s - r$ or $v - r$ depending on whether or not s is observed) exceeds the threshold $p_1(v) \equiv (1 - F(v))/f(v)$. Second, it is optimal to set $a(v)$ equal to 1 when the term multiplying $a(v)$ is greater than the term multiplying $(1 - a(v))$, that is,

$$\int \max\{v + s - r - p_1(v), 0\}\, dG(s) - K \geq \max\{v - r - p_1(v), 0\}, \qquad (7)$$

and set $a(v)$ to zero otherwise. This means that the consultant uncovers s whenever the expected virtual surplus with s known exceeds the virtual surplus with s not known. It turns out that there is a unique $\underline{v}_1$, characterized in the proof of Proposition 1, such that (7) holds if and only if $v \geq \underline{v}_1$, that is, $a(v) = \mathbf{1}_{v \geq \underline{v}_1}$, where $\mathbf{1}$ is the indicator function.

FIGURE 2

THE PARTICIPATION CONSTRAINT DOES NOT BIND AT $v = 0$

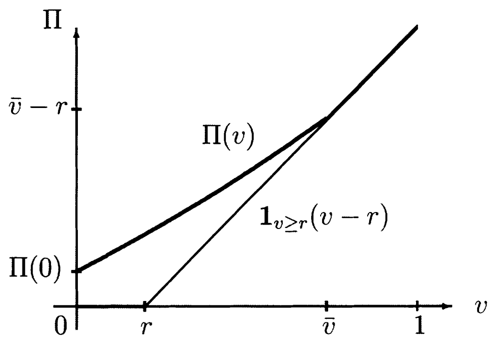
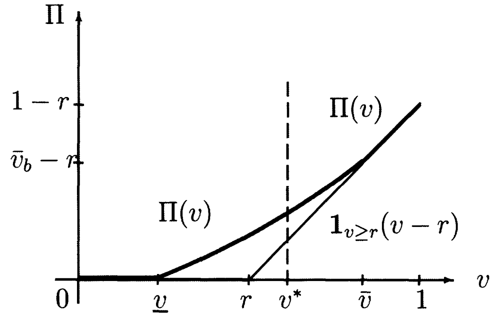


FIGURE 3

THE PARTICIPATION CONSTRAINT BINDS AT $v = 0$ AND $v = 1$



Under what conditions does Case 1 apply? The formula (6) is always valid. However, we maximized it pointwise while ignoring the incentive compatibility and the participation constraints. The functions a, x, and $\bar{x}$ determined above imply that $X(v) = 1 - G\,(r + (1 - F(v))/f(v) - v)$ if s is observed and zero otherwise.[18] Hence, X is weakly increasing, and the mechanism is incentive compatible by Lemma 1. We claim that the participation constraint holds if, and only if,

$$\int_{\underline{v}_1}^{1} 1 - G\left(r + \frac{1 - F(v)}{f(v)} - v\right) dv \geq 1 - r. \tag{8}$$

Notice that by Lemma 1, this is exactly the participation constraint of a client with type $v = 1$. Observe, however, that if the participation constraint holds for the highest type, then it holds for all types. The reason is the following. First, because $\Pi(0) = 0$ and the slope of Π (weakly) exceeds zero, the participation constraint holds whenever $v \leq r$. Second, suppose that the participation constraint is violated at some $v \in [r, 1]$. The slope of Π is X (by Lemma 1), which does not exceed one. On the other hand, the slope of the outside option is exactly one on $[r, 1]$. Hence, it follows that the participation constraint is also violated at $v = 1$, a contradiction. Therefore, Case 1 applies whenever (8) is satisfied.

Case 2. Suppose that in the optimal contract the highest type's participation constraint binds but the lowest type's does not, as in Figure 2. Again, from Lemma 1, by integration by parts we get

$$\int_0^1 \Pi(v)\,dF(v) = \Pi(1) - \int_0^1 F(v)X(v)\,dv.$$

This, combined with equations (3) and (5), yields for the consultant's expected payoff,

$$\overline{W} = \int_0^1 \left\{a(v)\left[\int \left(v + s - r + \frac{F(v)}{f(v)}\right)x(v, s)\,dG(s) - K\right] + (1 - a(v))\left(v - r + \frac{F(v)}{f(v)}\right)\bar{x}(v)\right\} dF(v) - \Pi(1). \tag{9}$$

This is just an alternative way to write (6). Instead of integrating from zero with $\Pi(0)$ as the integration constant, we integrate from one using $\Pi(1)$ as the constant.

In order to maximize $\overline{W}$, we set $\Pi(1) = 1 - r$ to make the participation constraint bind for the highest type. Similarly to Case 1, we maximize $\overline{W}$ pointwise by setting $x(v, s) = 1$ if $v + s + F(v)/f(v) \geq r$ and zero otherwise, and $\bar{x}(v) = 1$ if $v + F(v)/f(v) \geq r$ and zero

[18] We show in the proof of Proposition 1 that if $v \leq \underline{v}_1$, then $v - (1 - F(v))/f(v) < r$, hence the client does not execute the project.

otherwise. That is, the client undertakes the project whenever its expected net value exceeds $p_0(v) \equiv -F(v)/f(v)$. Furthermore, we set $a(v) = 1$ whenever

$$\int \max\{v + s - r - p_0(v), 0\}\, dG(s) - K \geq \max\{v - r - p_0(v), 0\}, \tag{10}$$

and $a(v) = 0$ otherwise. We show in the proof of Proposition 1 that there exists a unique $\bar{v}_0$ such that $a(v) = \mathbf{1}_{v \leq \bar{v}_0}$.

Under what conditions does Case 2 apply? The functions a, x, and $\bar{x}$ determined above imply that $X(v) = 1 - G(r - F(v)/f(v) - v)$ if s is observed and zero otherwise.[19] Hence, X is weakly increasing, and the mechanism is incentive compatible by Lemma 1. The participation constraint holds if, and only if,

$$\bar{v}_0 - \int_0^{\bar{v}_0} \left[1 - G\left(r - \frac{F(v)}{f(v)} - v\right)\right] dv \geq r. \tag{11}$$

Notice that by Lemma 1, this is exactly the participation constraint of a buyer with type zero (the left-hand side is $\Pi(0) + r$). However, if the participation constraint holds for $v = 0$, then it holds for all v. If $v \in [r, 1]$, then the participation constraint holds because $\Pi(1) = 1 - r$, and the slope of Π is less than one. If the participation constraint was violated at $v \in [0, r]$, it would also be violated at zero, because the slope of Π is X (by Lemma 1), which is weakly greater. Therefore Case 2 applies whenever (11) is satisfied.

Case 3. Finally, suppose that the participation constraint binds at both ends of the unit interval as in Figure 3.

Notice that in Figure 3, for any $v^* \in (\underline{v}, \bar{v})$, the indirect profit function on $[0, v^*]$ resembles the solution in Case 1 (depicted in Figure 1) in that the participation constraint binds for the lowest type, but does not bind for the highest type. Moreover, Π on $[v^*, 1]$ resembles the solution in Case 2, seen in Figure 2, in that the participation constraint does not bind for the lowest type but binds for the highest type. In the Appendix we show that in fact, for a *suitably chosen* v^*, the overall solution (the decision rule and the client's profit as functions of v) coincides with the optimal contracts conditional on $v \leq v^*$ and $v \geq v^*$, respectively. In words, the optimal mechanism in Case 3 is *as though* the consultant knew whether $v \leq v^*$ or $v \geq v^*$ and offered the optimal contract to the client contingent on which interval his type lies in.

Let us determine the conditional optimal contracts. The distribution of v conditional on $v \leq v^*$ is $F_L(v) = F(v)/F(v^*)$ with density $F_L(v) = F(v)/F(v^*)$. Therefore, in the optimal contract conditional on $v \leq v^*$, we have $x(v, s) = 1$ whenever $v + s - r \geq (1 - F_L(v))/f_L(v)$, and $\bar{x}(v) = 1$ whenever $v - r \geq (1 - F_L(v))/f_L(v)$. Notice that

$$\frac{1 - F_L(v)}{f_L(v)} = \frac{F(v^*) - F(v)}{f(v)};$$

therefore the client with type $v \in [0, v^*]$ undertakes the project whenever the project's net value is greater than the threshold $p_{v^*}(v) = (F(v^*) - F(v))/f(v)$. As in Case 1, the consultant sets $a(v) = \mathbf{1}_{v \geq \underline{v}}$, where the threshold $\underline{v}$ depends on the value of v^*.

In the optimal contract conditional on $v \in [v^*, 1]$, as in Case 2, the consultant finds out the realization of s if and only if $v \leq \bar{v}$ (where the threshold $\bar{v}$ depends on the value of v^*). Then, she directs the client to undertake the project whenever $v + s - r$ or $v - r$ (depending on whether or not s has been observed) exceeds $-F_H(v)/f_H(v)$, where $F_H(v) = (F(v) - F(v^*))/(1 - F(v^*))$ is the distribution of v conditional on $v \geq v^*$, and $f_H(v) = f(v)/(1 - F(v))$ is the corresponding density. Notice that

$$-\frac{F_H(v)}{f_H(v)} = \frac{F(v^*) - F(v)}{f(v)},$$

[19] We show in the proof of Proposition 1 that if $v \geq \bar{v}_0$, then $v - F(v)/f(v) > r$; hence the client executes the project.

that is, the client undertakes the project under exactly the same criterion as in the case when $v \leq v^*$.

To summarize: in Case 3, the optimal contract is such that the consultant finds out the realization of s (at cost K) whenever $v \in [\underline{v}, \bar{v}]$, and the client undertakes the project if and only if the virtual surplus,

$$v + s - r - \frac{F(v^*) - F(v)}{f(v)} - K \text{ or } v - r - \frac{F(v^*) - F(v)}{f(v)},$$

depending on whether or not s is observed, is nonnegative. The value of v^* is determined so that the client's profit at $v = v^*$ is the same in the optimal contract conditional on $v \leq v^*$ and in the optimal contract conditional on $v \geq v^*$. The boundary values $\underline{v}$ and $\bar{v}$ are such that for all $v \in [\underline{v}, \bar{v}]$, the expected virtual surplus when observing s exceeds the virtual surplus when not observing it,

$$\int \max\{v + s - r - p_{v^*}(v), 0\}\, dG(s) - K \geq \max\{v - r - p_{v^*}(v), 0\},$$

where $p_{v^*}(v) = (F(v^*) - F(v))/f(v)$.

We summarize our findings in the following proposition.

Proposition 1. If the consultant can find out the realization of s at a cost K, then the optimal mechanism, $\{a, x, \bar{x}, \Pi\}$, is as follows. There exist parameters b, $\underline{v}_b$, $\bar{v}_b \in [0, 1]$ such that $a(v) = \mathbf{1}_{v \in [\underline{v}_b, \bar{v}_b]}$, $x(v, s) = \mathbf{1}_{v+s \geq r+p(v)}$, and $\bar{x}(v) = \mathbf{1}_{v \geq r+p(v)}$, where $p(v) = (b - F(v))/f(v)$. Π satisfies (4), where X is defined by (3). Moreover, if $\underline{v}_b > 0$ then $\Pi(\underline{v}_b) = 0$, otherwise $\bar{v}_b < 1$ and $\Pi(\bar{v}_b) = 1 - r$.

Proof. See the Appendix, where b, $\underline{v}_b$, and $\bar{v}_b$ are determined as well.

Remark 1. If $r > 1$, then $b = 1$ and $\bar{v}_1 = 1$ (Case 1 applies), whereas if $r < 0$, then $b = 0$ and $\underline{v}_0 = 0$ (Case 2 applies). It can also be verified that if $K = 0$, then $\underline{v}_b = 0$ and $\bar{v}_b = 1$. That is, if uncovering the realization of s is costless, then all types of the client are served by the consultant.

Note (by inspecting Figures 1–3) that for client types outside the interval $[\underline{v}_b, \bar{v}_b]$, the participation constraint binds, $\Pi(v) = \max\{v - r, 0\}$. Because the consultant does not provide any service (or generate surplus) for types $v \notin [\underline{v}_b, \bar{v}_b]$, these types act as though they did not contract with the consultant: types $v < \underline{v}_b$ do not undertake the project, whereas types $v > \bar{v}_b$ do.[20] In the optimal mechanism of Proposition 1, if the consultant chooses not to find out the value of s, then she does not introduce distortion in the sense that the client is asked to undertake a project whenever $v > r$, which is the socially optimal decision conditional on s not being revealed.

□ **The consultant cannot observe s.** Our main result is that the optimal mechanism in the case where $z = s$ (as described in Proposition 1) can be implemented by the consultant even if she *cannot observe* the realization of s at all when disclosing it to the client, that is, if z and s are independent. Therefore, the solution to the relaxed problem studied in the previous section can be implemented irrespective of the correlation between z and s.

Recall from Proposition 1 that when $z = s$, in the optimal mechanism, a client with type $v \in [\underline{v}_b, \bar{v}_b]$ (for whom the consultant uncovers the realization of s) undertakes the project whenever his net profit from the project, $v + s - r$, is greater than $p(v) = (b - F(v))/f(v)$. Notice that the same decision rule could potentially be implemented by allowing the client with type $v \in [\underline{v}_b, \bar{v}_b]$ to learn s and asking him to make an additional payment of $p(v)$ in case he undertakes the project. If there was an incentive-compatible mechanism that assigned such a conditional payment $p(v)$ to type v, then the client would undertake the project if and only if $v + s - r - p(v)$ exceeded 0. The key observation here is that the consultant need not observe s for such an implementation,

[20] Comparing this with Proposition 1, we conclude that $v < r + p(v)$ for all $v < \underline{v}_b$, whereas $v > r + p(v)$ for all $v > \bar{v}_b$.

as long as an appropriate incentive-compatible mechanism can be found. In the mechanism of Proposition 1, client types outside the interval $[\underline{v}_b, \bar{v}_b]$ do not get to find out s. The decision rule allows them to undertake the project whenever they find it profitable without help from the consultant, that is, if $v - r \geq 0$. An easy way to implement this part of the decision rule is by simply not offering those types a contract.

In order to generate the same profit for the consultant as in the relaxed problem (while implementing the same decision rule and total surplus), we make the client's profit the same as in the mechanism of Proposition 1. We do so by requiring client type $v \in [\underline{v}_b, \bar{v}_b]$ to pay an unconditional ("upfront") fee of $c(v)$ such that

$$\Pi(v) = \int \max\{v + s - r - p(v), 0\}\, dG(s) - c(v), \tag{12}$$

where $\Pi(v)$ is the client's payoff in the mechanism of Proposition 1.

The question is whether or not this mechanism (determined by the interval $[\underline{v}_b, \bar{v}_b]$ and the functions p and c) is incentive compatible, that is, whether or not the client reports his type truthfully. Clearly, when s is not observed by the consultant, the mechanism has to satisfy additional incentive compatibility constraints compared to the case where s is observed by her. The following proposition states that all these constraints are satisfied in the mechanism proposed above.

Proposition 2. The mechanism described in Proposition 1 (the optimal contract when $z = s$) can be implemented even if the consultant cannot directly observe s as she discloses it to the client (that is, when z and s are independent).

Irrespective of the correlation of z and s, the optimal mechanism can be implemented as follows. The client reports his type, v. If $v \notin [\underline{v}_b, \bar{v}_b]$, then the consultant does not provide advice. If $v \in [\underline{v}_b, \bar{v}_b]$, then the client pays the consultant an upfront fee of $c(v)$, and the realization of s is revealed to him. If he then chooses to undertake the project, he pays an additional premium of $p(v)$, where $p(v) = (b - F(v))/f(v)$.

The parameters b, $\underline{v}_b$, and $\bar{v}_b$ are the same as in Proposition 1.

Proof of Proposition 2. Let $p(v) = (b - F(v))/f(v)$ and define $c(v)$ according to equation (12) with b, $\underline{v}_b$, $\bar{v}_b$, and Π as in Proposition 1. Note that Π satisfies the conditions of Lemma 1, that is, $\Pi(v') = \Pi(v) + \int_v^{v'} X(z)\, dz$, where

$$X(v) = \begin{cases} 0 & \text{for } v < \underline{v}_b, \\ 1 - G(r + p(v) - v) & \text{for } v \in [\underline{v}_b, \bar{v}_b], \\ 1 & \text{for } v > \bar{v}_b. \end{cases} \tag{13}$$

We claim that the mechanism is incentive compatible even if only the client observes the realization of s.

Note that if $v' \notin [\underline{v}_b, \bar{v}_b]$ is reported, then the payoff of type v is $\max\{v - r, 0\}$, which does not exceed $\Pi(v)$, as the participation constraint is satisfied in the mechanism of Proposition 1. Therefore, no type has an incentive to misreport $v' \notin [\underline{v}_b, \bar{v}_b]$.

Suppose now that a client with type v reports $v' \in [\underline{v}_b, \bar{v}_b]$. We consider the case $v \in [\underline{v}_b, \bar{v}_b]$; the proof for $v \notin [\underline{v}_b, \bar{v}_b]$ is essentially identical and therefore omitted. Recall that

$$\Pi(v') = \Pi(v) + \int_v^{v'} (1 - G(r + p(z) - z))\, dz. \tag{14}$$

(We use the convention $\int_a^b = -\int_b^a$ for $a < b$ throughout.) If type v reports v' then he eventually undertakes the project if and only if $v + s \geq r + p(v')$, and his deviation payoff is

$$\pi(v, v') = \int \max\{v + s - r - p(v'), 0\}\, dG(s) - c(v').$$

By (12), we have

$$\Pi(v') - \pi(v, v') = \int (\max\{v' + s - r - p(v'), 0\} - \max\{v + s - r - p(v'), 0\})\, dG(s).$$

By the Fundamental Theorem of Calculus,

$$\max\{v' + s - r - p(v'), 0\} - \max\{v + s - r - p(v'), 0\} = \int_v^{v'} \mathbf{1}_{z+s-r-p(v')}\, dx.$$

Substitute this back into the previous equation to get

$$\Pi(v') - \pi(v, v') = \int \int_v^{v'} \mathbf{1}_{s+z-r-p(v')}\, dx \;\; dG(s).$$

By changing the order of integration, we find

$$\Pi(v') - \pi(v, v') = \int_v^{v'} \int \mathbf{1}_{s+z-r-p(v')}\, dG(s)\, dx = \int_v^{v'} [1 - G(r + p(v') - z)]\, dz. \quad (15)$$

In order to establish incentive compatibility, we need to show $\Pi(v') - \pi(v, v') \geq \Pi(v') - \Pi(v)$. Using (14) and (15), this inequality can be rewritten as

$$\int_v^{v'} [1 - G(r + p(v') - z)]\, dz \geq \int_v^{v'} (1 - G(r + p(z) - z))\, dz. \quad (16)$$

By the log concavity of v's density, $p(v) = (b - F(v))/f(v)$ is decreasing in v for any $b \in [0, 1]$, and so (16) holds.

We have implemented the optimal allocation rule in a direct mechanism where the client reports his type and the consultant offers a contract conditional on the report. Of course, the same allocation can also be implemented by simply offering a menu of contracts from which the client can choose one. More precisely, the consultant can offer the menu $\{(c(v), p(v))\}_{v \in [\underline{v}_b, \bar{v}_b]}$ to each client. Then the client decides whether to pick an item or not to contract with the consultant. If he picks the pair $(c(v), p(v))$, then he has to pay $c(v)$ upfront, gets to learn s, and pays an additional $p(v)$ if the project is undertaken.

A feature of the menu $\{(c(v), p(v))\}_{v \in [\underline{v}_b, \bar{v}_b]}$ is that whereas p is a decreasing function, c is an increasing one. This attribute helps to understand how such a menu can be used to sort the clients according to their types. The larger the type of client, the more confident he is that the project is worth undertaking. This, in turn, implies that he is more likely to pay the additional premium p. As a result, he is willing to pay more upfront to reduce the premium by a dollar than a client with a smaller type would.

The solutions in the special cases when $r > 1$ and $r < 0$, respectively, are quite insightful and interesting on their own. If $r > 1$, then the value of the client's outside option is zero for all types. Therefore the solution is like the one depicted in Figure 1: the participation constraint binds for the lowest type and not for the highest type, and $b = 1$. When $r > 1$, the client is originally "pessimistic" in the sense that his estimate regarding the profitability of the project is always nonpositive. He would never undertake the project without the consultant's advice (i.e., without learning that s is sufficiently large and positive). Note that in this case, the client has to pay the consultant if he decides to undertake the project: $p = (1 - F)/f > 0$ for all v. If $r \leq 0$, the client is always "optimistic," $v > r$ for all v, and $p = -F/f < 0$. That is, when $r \leq 0$, the client pays more to the consultant in the case where he does not undertake the project than in the case he does.

In these special cases, the client has to make a net payment to the consultant *when the consultant's advice makes the client change his mind*: if he undertakes the project while $v \leq r$ for all v, or if he does not undertake the project while $v \geq r$ for all v. Of course, in general (e.g., when $r \in [0, 1]$), the consultant does not know *a priori* which of the two actions of the client signifies that he has changed his mind.

□ **Necessary and sufficient conditions.** A natural question to ask is: Can the optimal allocation of the case where $z = s$ always be implemented in the case where z and s are independent? In other words, is it true that the consultant can never gain more by *directly* observing s? We show below that the answer is no, the two problems are generally not equivalent.

In what follows, we fix the parameters r, K, and the cdf G in our model. We shall characterize the necessary and sufficient conditions on the distribution of the types F, under which the allocation described in Proposition 1 can be implemented in the case where $z = s$ and in the case where z and s are independent. In the independent case, we restrict attention to mechanisms in the form of menus $(c(v), p(v))_{v\in[0,1]}$. In the discussion paper version of our article (Eső and Szentes, 2004), this restriction is shown to be without loss of generality.

The $z = s$ case. Recall from Lemma 1 that a necessary and sufficient condition for the allocation described in Proposition 1 to be implementable is that the function X is weakly increasing. The function X, defined by (13), is obviously weakly increasing on $[0, \underline{v}_b]$ and on $[\bar{v}_b, 1]$, and is continuous at $\underline{v}_b$ and $\bar{v}_b$. Therefore, the optimal allocation is implementable if and only if

$$v' \geq v \Leftrightarrow 1 - G(r + p(v') - v') \geq 1 - G(r + p(v) - v). \tag{17}$$

That is, the necessary and sufficient condition is that $v - (b - F(v))/f(v)$ is weakly increasing on $[\underline{v}_b, \bar{v}_b]$. This follows from the assumption that f is log-concave (which implies that p is decreasing). Note that even if p is increasing, however, the necessary-sufficient condition can still hold. Therefore the log-concavity assumption is sufficient but not necessary.

The independent case. In the proof of Proposition 2, we showed that the necessary and sufficient condition for the optimal allocation to be implementable when z and s are independent is (16), which is equivalent to

$$v' \geq v \Leftrightarrow 1 - G(r + p(v') - v) \geq 1 - G(r + p(v) - v), \tag{18}$$

for all $v, v' \in [\underline{v}_b, \bar{v}_b]$. This implies that the assumption that p is increasing on $[\underline{v}_b, \bar{v}_b]$ is not only sufficient but also necessary in this case.

Clearly, condition (18) is stronger than condition (17). What is the reason for the discrepancy? Recall that in the case where $z = s$, if a client reports type v' and the realization of the consultant's signal is s, he executes the project if and only if $v' + s - p(v')$ is larger than r. That is, the decision whether or not to execute the project does not depend on his true type. In the case where z and s are independent, however, the consultant does not observe the realization of s, and hence she is unable to force the client to act according to the reported type. A client who misreports his type in the first stage of the mechanism can and will adjust his decision regarding the project accordingly. If client type v reports v', he only implements the project if $v + s - p(v')$ is larger than r. This implies that, in general, a deviator in the model where s is not observed by the consultant can do better than a deviator in the model where $z = s$ because *he* makes the decision regarding the project. This explains why (18) is stronger than (17).

4. Conclusions

■ We analyzed a model of the advisor–client relationship where the role of the advisor is to disclose "clues" to the client that only he (the client) can understand perfectly. These clues, or signals, refine the client's original private estimate regarding the profitability of the client's project. We assumed that the client's action (whether or not he undertakes the project) is contractible, and therefore the consultant can offer a deal where the client pays her differently depending on whether he undertakes the project upon evaluating her advice. We derived the consultant's optimal contract, which can be thought of as a menu of such transfer pairs. Some items on the menu may require the client to pay more if he undertakes the project, other items may require higher payments if he does not. The consultant discloses the additional signals only if the client agrees to one of the items.

In general, in the optimal contract, only clients with value estimates between certain thresholds take up the consultant's offer. Among those who do, clients with higher estimates choose transfer pairs where the signed difference between what they have to pay upon undertaking the project and upon not undertaking it is smaller. In interesting special cases of the model, the optimal contract can be interpreted as one where the client pays the consultant more whenever her advice has made him change his mind as to whether to undertake the project.

The most interesting finding, we believe, is that no matter how precisely the consultant can observe the impact of her advice on the client's objectives, her payoff in the optimal contract is the same as though she knew *exactly* how the client's value estimate changed by her signals or clues. Even if the consultant is ignorant regarding how her advice affects her client, as long as she has the power to design their contract and can condition it on the decision of the client, she can do just as well as if she understood the precise effect of her advice.

Appendix

■ **The proof of Proposition 1.** Assume that $x(v, s) = \mathbf{1}_{v+s-(b-F(v))/f(v)>r}$. Then, we can compute the smallest and largest types, $\underline{v}_b$ and $\bar{v}_b$, respectively, for which the virtual surplus conditional on the disclosure of s exceeds the virtual surplus conditional on not disclosing s. Formally, for all $b \in [0, 1]$, define

$$\underline{v}_b = \min\left\{ v \in [0,1] : \int_{r+\frac{b-F(v)}{f(v)}-v}^{\infty} \left(v + s - r - \frac{b - F(v)}{f(v)} \right) dG(s) \geq K \right\}, \tag{A1}$$

$$\bar{v}_b = \max\left\{ v \in [0,1] : \int_{-\infty}^{r+\frac{b-F(v)}{f(v)}-v} \left(r + \frac{b - F(v)}{f(v)} - v - s \right) dG(s) \geq K \right\}. \tag{A2}$$

Lemma A1. For all $b \in [0, 1]$, $\underline{v}_b < \bar{v}_b$ are well defined by (A1) and (A2); moreover, $F(\underline{v}_b) \leq b \leq F(\bar{v}_b)$, and both $\underline{v}_b$ and $\bar{v}_b$ are continuous and weakly increasing in b with $\underline{v}_0 = 0$ and $\bar{v}_1 = 1$.

Proof. To see existence, let $v^* = F^{-1}(b)$, that is, $F(v^*) = b$. The left-hand side of the inequality in (A1) is continuous and strictly increasing in $\underline{v}_b$. At $\underline{v}_b = v^*$, the expression becomes $\int_{r-v^*}^{\infty} (v^* + s - r)\, dG(s)$, which equals $w(v^*) + \mathbf{1}_{v^*>r}(v^* - r)$ by (2). However, $w(v^*) > K$ by assumption, so indeed there exists $\underline{v}_b$ such that (A1) holds. Similarly, the left-hand side of the inequality in (A2) is continuous and strictly decreasing in $\bar{v}_b$. At $\bar{v}_b = v^*$, it becomes $\int_{-\infty}^{r-v^*} (r - v^* - s)\, dG(s) = \int_{r-v^*}^{\infty} (v^* + s - r)\, dG(s) + r - v^*$, which equals $w(v^*) + \mathbf{1}_{r>v^*}(r - v^*)$ by (2), and $w(v^*) > K$ by assumption. From this argument, it is also clear that $F(\underline{v}_b) \leq b \leq F(\bar{v}_b)$, which then implies $\underline{v}_0 = 0$ and $\bar{v}_1 = 1$.

It is easy to see that $\underline{v}_b$ and $\bar{v}_b$ are continuous in b (no matter what the distribution of s is). Because the integral in (A1) is strictly decreasing in b, and the integral in (A2) is strictly increasing in b, both $\underline{v}_b$ and $\bar{v}_b$ are weakly increasing in b.

Next, we define b depending on the value of r. If (8) holds then $b = 1$, and if (11) holds then $b = 0$. Otherwise, let b any solution to

$$\bar{v}_b - r - \int_{\underline{v}_b}^{\bar{v}_b} \left[1 - G\left(r + \frac{b - F(v)}{f(v)} - v \right) \right] dv = 0. \tag{A3}$$

Intuitively, b is defined such that a client with type $\underline{v}_b$, as well as with type $\bar{v}_b$, gets exactly his outside option. That is, these types are indifferent between accepting and rejecting the contract of the consultant.

In order to see that there exists $b \in [0, 1]$ satisfying (A3) if (8) and (11) do not hold, first note that the left-hand side of (A3) is continuous in b. If (11) does not hold, then this expression is positive at $b = 1$ (as $\bar{v}_1 = 1$ by Lemma A1). If (8) does not hold, then the same expression is negative at $b = 0$ (as $\underline{v}_0 = 0$ by Lemma A1). Therefore, if neither (11) nor (8) holds then, by the Intermediate Value Theorem, there exists a $b \in (0, 1)$, not necessarily unique, that satisfies (A3).

Proof of Proposition 1. We prove the proposition according to Cases 1–3.

Case 1. We have established in the text that if (8) holds, then $b = 1$. It remains to show that (i) if $v \leq \underline{v}_1$ then $v - (1 - F(v))/f(v) < r$, and (ii) $a(v)$ defined by (7) coincides with $\mathbf{1}_{v \geq \underline{v}_1}$. Notice that $\underline{v}_1 \leq r$ must hold because the left-hand side of the inequality in (8) does not exceed $1 - \underline{v}_1$. Furthermore, $v - (1 - F(v))/f(v)$ is increasing in v, hence (i) follows. To see (ii), first consider the interval $[0, \underline{v}_1)$. As $\underline{v}_1 \leq r$, the right-hand side of (7) is zero, whereas the left-hand side is weakly positive by (A1). Hence, if $v < \underline{v}_1$ then the inequality in (A1) does not hold, and therefore (7) does not hold either, implying that $a(v) = 0$ if $v \leq \underline{v}_1$. If $v \in (\underline{v}_1, r)$, then the right-hand side of (7) is zero, but the left-hand side is positive, because the integrand in (7) is increasing in v. This shows that $a(v) = 1$ on $v \in (\underline{v}_1, r)$. Finally, $a(v) = 1$ on $v \in [r, 1]$ follows from the assumption that $w(v) > K$ for all v.

Case 2. We have also established in the text that if (11) holds, then $b = 0$. It remains to show that (i) if $v \geq \bar{v}_0$ then $v + F(v)/f(v) > r$, and (ii) $a(v)$ defined by (10) coincides with $\mathbf{1}_{v \leq \bar{v}_0}$. Observe that $\bar{v}_0 \geq r$ because the left-hand side of the inequality in (11) is nonnegative. In addition, $v + F(v)/f(v)$ is increasing in v, and hence (i) follows. To see (ii), first notice that the inequality in (A2) can be rewritten for $b = 0$ as

$$\int_{r+p_0(v)-v} v + s - r - p_0(v)\, dG(s) - K \geq v - r - p_0(v). \tag{A4}$$

Consider first a $v \in (\bar{v}_0, 1]$. Because $\bar{v}_0 \geq r$, (10) coincides with (A4). By the definition of $\bar{v}_0$, (10) cannot hold, and hence $a(v) = \mathbf{1}_{v \leq \bar{v}_0} = 0$ on $(\bar{v}_0, 1]$. Suppose now that $v \in [r, \bar{v}_0,]$. On this interval, the right-hand side of (10) is still $v - r - p_0(v)$. Furthermore, the left-hand side of (A4) increases in v slower than the right-hand side. Because (A4) holds for $\bar{v}_0$ it must also hold for $v \in [r, \bar{v}_0,]$, showing that $a(v) = \mathbf{1}_{v \leq \bar{v}_0} = 1$ on this interval. If $v < r$, then $a(v) = \mathbf{1}_{v \leq \bar{v}_0}$ follows from $w(v) > K$.

Case 3. Suppose that (A3) holds, and hence $b \in (0, 1)$. First we establish that the mechanism proposed in the proposition is incentive compatible. The function X induced by the mechanism is

$$X(v) = \begin{cases} 0 & \text{for } v < \underline{v}_b, \\ 1 - G\left(r + \frac{b-F(b)}{f(b)} - v\right) & \text{for } v \in [\underline{v}_b, \bar{v}_b], \\ 1 & \text{for } v > \bar{v}_b. \end{cases}$$

Note that $p = (b - F)/f$ is weakly decreasing by the log concavity of f, and hence incentive compatibility follows from Lemma 1. Next we show that the proposed mechanism satisfies the participation constraint by setting $\Pi(0) = 0$. Note that $\underline{v}_b < r < \bar{v}_b$ because the integrand in (A3) is always between zero and one, and hence the value of the integral is between zero and $\bar{v}_b - \underline{v}_b$. (4) implies that $\Pi(v) = 0$ for all $v \in [0, \underline{v}_b]$. Combining (4) and (A3), we get $\Pi(\bar{v}_b) = \bar{v}_b - r$, which implies $\Pi(v) = v - r$ for all $v \in [\bar{v}_b, 1]$. Finally, because $X \in (0, 1)$,

$$\Pi(v) > \max\{0, v - r\} \quad \text{for all } v \in (\underline{v}_b, \bar{v}_b). \tag{A5}$$

The optimality of the mechanism is established in the text. The type v^* is defined such that $F(v^*) = b$. It remains to show that (i) the payoff of the client with type v^* is the same in the optimal contract conditional on $v \leq v^*$ and in the optimal contract conditional on $v \geq v^*$, and (ii) $a(v) = 1_{v \in [\underline{v}_b, \bar{v}_b]}$. The proof of (ii) is essentially identical to the proofs in Cases 1 and 2, hence we omit it. To see (i), first notice that because $F(\underline{v}_b) \leq b < F(\bar{v}_b)$ by Lemma A1, we have $\underline{v}_b \leq v^* \leq \bar{v}_b$. Note that

$$\frac{b - F(v)}{f(v)} = \begin{cases} \dfrac{1 - F_L(v)}{f_L(v)} & \text{for all } v \in [0, v^*], \\[2ex] \dfrac{-F_H(v)}{f_H(v)} & \text{for all } v \in [v^*, 1]. \end{cases}$$

By $\Pi(\underline{v}_b) = 0$, $\Pi(\bar{v}_b) = \bar{v}_b - r$, (4), and (A3),

$$\int_{\underline{v}}^{v^*} \left[1 - G\left(r + \frac{1 - F_L(v)}{f_L(v)} - v\right)\right] dv = \bar{v} - r - \int_{v^*}^{\bar{v}} \left[1 - G\left(r - \frac{F_H(v)}{f_H(v)} - v\right)\right] dv.$$

Notice that the left-hand side is the payoff of the client with type v^* in the optimal contract conditional on $v \leq v^*$, and the right-hand side is his payoff in the optimal contract conditional on $v \geq v^*$.

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